

Math 182
25 April 2017

Quiz 6
Show Appropriate Work

Name: _____
Point Values in .

1. 2 Find parametric equations, $x(t)$ and $y(t)$, for the line segment from $(4, 2)$ to $(1, 0)$. Include the domain of t .

$$\begin{aligned}x(t) &= 4 - 3t & 0 \leq t \leq 1 \\y(t) &= 2 - 2t\end{aligned}$$

2. 2 Find parametric equations, $x(t)$ and $y(t)$, for the circle with center $(3, -2)$ and radius 5. Include the domain of t .

$$\begin{aligned}x(t) &= 5 \cos t + 3 & 0 \leq t \leq 2\pi \\y(t) &= 5 \sin t - 2\end{aligned}$$

3. Consider the path given by $c(t) = (t^2 + 3, 1 - t^3)$.

- (a) 4 Find the length of the path for $t \in [0, 1]$.

$$\begin{aligned}x' &= 2t & s &= \int_0^1 t \sqrt{4+t^2} dt &= \frac{1}{18} \int_4^{13} u^{1/2} du \\y' &= -3t^2 & u &= 4+t^2 &= \frac{1}{18} \cdot \frac{2}{3} u^{3/2} \\ds &= \sqrt{4t^2+1t^4} dt & du &= 18t dt &= \frac{1}{27} \left(13^{3/2} - 8 \right) \\&\Rightarrow t \sqrt{4+t^2} dt & 18t &\longmapsto u &= \frac{1}{27} (13^{3/2} - 8)\end{aligned}$$

- (b) 1 Find the speed, $\frac{ds}{dt}$, along the path at $t = 1$.

$$\left. \sqrt{4t^2+1t^4} \right|_{t=1} = \sqrt{13}$$

4. 2 Convert the following polar equation into rectangular coordinates.

$$r = \frac{6}{2 \cos \theta - 3 \sin \theta}$$

$$2r \cos \theta - 3r \sin \theta = 6$$

$$2x - 3y = 6$$

You may find the following identities useful:

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \text{ and } \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

5. [5] Find the area inside $r = 2 \cos 3\theta$ and outside $r = \sqrt{3}$, the shaded area in the figure.

Intersection when $2 \cos 3\theta = \sqrt{3}$

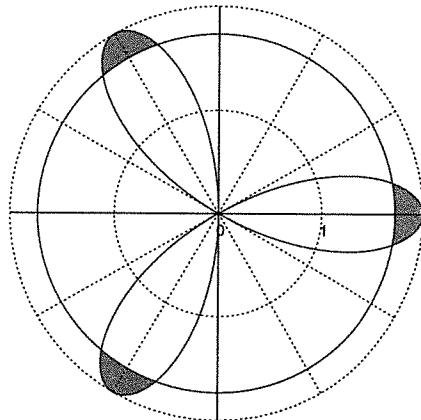
$$\cos 3\theta = \frac{\sqrt{3}}{2}$$

$$3\theta = \frac{\pi}{6} \quad \text{so } \theta = \frac{\pi}{18}$$

$$6 \cdot \frac{1}{2} \int_0^{\pi/18} [4 \cos^2 3\theta - 3] d\theta$$

$$= 3 \int_0^{\pi/18} [2 + 2 \cos 6\theta - 3] d\theta$$

$$= 3 \left(-\theta + \frac{1}{3} \sin 6\theta \right) \Big|_0^{\pi/18} = 3 \left(-\frac{\pi}{18} + \frac{1}{3} \cdot \frac{\sqrt{3}}{2} \right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$$



6. [4] Find the length of the polar curve $r = (\theta^2 - 1)^{\frac{1}{2}}$ for $\theta \in [0, \pi]$.

$$r^2 = (\theta^2 - 1)^2 = \theta^4 - 2\theta^2 + 1$$

$$(r')^2 = (2\theta)^2 = 4\theta^2$$

$$r^2 + (r')^2 = \theta^4 + 2\theta^2 + 1 = (\theta^2 + 1)^2$$

$$ds = (\theta^2 + 1)d\theta$$

$$s = \int_0^{\pi} (\theta^2 + 1) d\theta = \frac{\theta^3}{3} + \theta \Big|_0^{\pi} = \frac{\pi^3}{3} + \pi$$