10.6 Problems

1. a) $g(x) = 2x \cdot \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n \cdot 2x = \sum_{n=0}^{\infty} (-1)^n 2 x^{n+1}$

$= 2x - 2x^2 + 2x^3 - 2x^4 + \cdots$

if $|-x| < 1 \Rightarrow |x| < 1$

b) $h(x) = \frac{2}{(1+x)^2} = \frac{d}{dx}\left(\frac{2x}{1+x}\right) = 2 - 4x + 6x^2 - 8x^3 + \cdots$

$= \sum_{n=0}^{\infty} (-1)^n 2(n+1) x^n$ if $|x| < 1$

c) $k(x) = \frac{2}{1+x}$, $c = 3$

$k(x) = \frac{2}{4 - -(x-3)} = \frac{2}{4\left(1 - -\left(\frac{x-3}{4}\right)\right)} = \sum_{n=0}^{\infty} \frac{1}{2} \cdot \left(-\left(\frac{x-3}{4}\right)\right)^n$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2} \frac{(x-3)^n}{4^n}$ if $\left|-\left(\frac{x-3}{4}\right)\right| < 1 \Rightarrow |x-3| < 4$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n(-2)^n}$

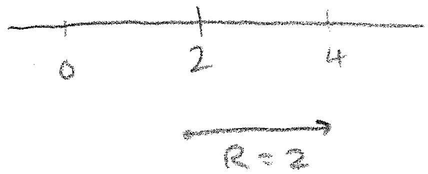
a) $\displaystyle\lim_{n\to\infty} \sqrt[n]{\left|\frac{(x-2)^n}{n(-2)^n}\right|} = \lim_{n\to\infty} \left| \frac{x-2}{n^{1/n}\cdot -2} \right| = \frac{|x-2|}{2}$ since $\displaystyle\lim_{n\to\infty} n^{1/n} = 1$

The series will converge if $\dfrac{|x-2|}{2} < 1$.

$\Rightarrow |x-2| < 2$

Thus, $R = 2$.

b) Find I (test endpoints): let $x = 4$: $\displaystyle\sum_{n=1}^{\infty} \frac{(4-2)^n}{n(-2)^n}$



$\displaystyle = \sum_{n=1}^{\infty} \frac{2^n}{n(-1)^n 2^n}$

$\displaystyle = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ alternating Harmonic converges

let $x = 0$: $\displaystyle\sum_{n=1}^{\infty} \frac{(0-2)^n}{n(-2^n)} = \sum_{n=1}^{\infty} \frac{1}{n}$, Harmonic, diverges.

Thus, $I = (0, 4]$.

c) $f'(x) = \frac{d}{dx}\left(\sum_{n=1}^{\infty} \frac{(x-2)^n}{n(-2)^n}\right) = \sum_{n=1}^{\infty} \frac{n(x-2)}{n(-2)^n} = \sum_{n=1}^{\infty} \frac{(x-2)^{n-1}}{(-2)^n}$

d) From a), $R=2$. Center is still 2, so we need only test endpoints.

<u>let $x=4$</u>: $\sum_{n=1}^{\infty} \frac{(4-2)^{n-1}}{(-2)^n} = \sum_{n=1}^{\infty} \frac{2^{n-1}}{(-1)^n 2^n} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{2}$, which diverges by Divergence Test ($\lim_{n\to\infty} (-1)^n \cdot \frac{1}{2}$ DNE).

<u>let $x=0$</u>: $\sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{(-2)^n} = \sum_{n=1}^{\infty} \frac{-1}{2}$, which diverges by Divergence Test ($\lim_{n\to\infty} \frac{-1}{2} \neq 0$).

Thus, $I = (0,4)$.

e) $F(x) = \int f(x)\,dx = \int \left(\sum_{n=1}^{\infty} \frac{(x-2)^n}{n(-2)^n}\right) dx = \sum_{n=1}^{\infty} \frac{(x-2)^{n+1}}{(n+1)\cdot n(-2)^n} + A$

$F(2)=0$, so $A=0$.

f) Again $R=2$, so we need only test endpoints.

<u>let $x=4$</u>: $\sum_{n=1}^{\infty} \frac{(2)^{n+1}}{(n^2+n)(-2)^n} = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2}{n^2+n}$, which converges by AST

$\left(\frac{2}{n^2+n} > 0,\ \lim_{n\to\infty} \frac{2}{n^2+n} = 0,\ \frac{2}{(n+1)^2+(n+1)} < \frac{2}{n^2+n}\right)$

<u>let $x=0$</u>: $\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{(n^2+n)(-2)^n} = \sum_{n=1}^{\infty} \frac{-2}{n^2+n}$. Consider $\sum_{n=1}^{\infty} \left|\frac{-2}{n^2+n}\right|$, which converges by Comp. Test

$\left(0 \le \frac{2}{n^2+n} < \frac{2}{n^2}\right)$

Thus, $I = [0,4]$.