Math 182 Problems
Section: 10.3
Due: 20 Mar 2018

Name: Key

Point values in $\boxed{\text{boxes}}$.

1. $\boxed{2.5}$ Use the Comparison Test to show $\sum_{n=2}^{\infty} \frac{\ln n}{n^2+1}$ converges.

For large n, $\ln n \le n^{1/2}$.

Thus, $0 \le \frac{\ln n}{n^2+1} \le \frac{n^{1/2}}{n^2} = \frac{1}{n^{3/2}}$

Because $\sum_{n=2}^{\infty} \frac{1}{n^{3/2}}$ converges ($p = 3/2 > 1$), then $\sum_{n=2}^{\infty} \frac{\ln n}{n^2+1}$ must converge by Comparison Test.

1. Inequality	2. Known Series	3. Conclusion	4. Logic	5. Notation

2. $\boxed{2.5}$ Use the Limit Comparison Test to show $\sum \frac{2n+3}{\sqrt[3]{n^5+7n^2+1}}$ diverges.

$a_n = \frac{2n+3}{\sqrt[3]{n^5+7n^2+1}} > 0$, $b_n = \frac{n}{n^{5/3}} = \frac{1}{n^{2/3}} > 0$

$$\lim_{n\to\infty} \frac{\frac{2n+3}{\sqrt[3]{n^5+7n^2+1}}}{\frac{1}{n^{2/3}}} = \lim_{n\to\infty} \frac{(2n+3)n^{2/3}}{\sqrt[3]{n^5+7n^2+1}} = \lim_{n\to\infty} \frac{2n^{5/3}+3n^{2/3}}{\sqrt[3]{n^5+7n^2+1}}$$
$$= 2 > 0$$

Because $\sum \frac{1}{n^{2/3}}$ diverges ($p = 2/3 < 1$), $\sum \frac{2n+3}{\sqrt[3]{n^5+7n^2+1}}$ must diverge by the Limit Comparison Test.

1. Limit	2. Known Series	3. Conclusion	4. Logic	5. Notation

3. Show $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges if and only if $p > 1$.

(a) For $p = 0$ the series is the Harmonic series and hence diverges.

(b) $\boxed{2.5}$ Use a comparison to show the series diverges for $p < 0$.

b) For large n, $0 \leq \frac{1}{n} \leq \frac{1}{n(\ln n)^p}$ (because $\ln n > 1$ when $n \geq 3$)

Because $\sum_{n=2}^{\infty} \frac{1}{n}$ diverges, $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ diverges (for $p<0$) by the Comparison Test.

1. Inequality	2. Known Series	3. Conclusion	4. Logic	5. Notation

(c) $\boxed{2.5}$ Use the Integral Test to test for $p > 0$.

let $f(x) = \frac{1}{x(\ln x)^p}$, which is positive and continuous for $x \geq 2$.

To show $f(x)$ is decreasing, we show $f'(x) < 0$:

$$f'(x) = \frac{0 - [x \cdot p(\ln x)^{p-1} \cdot \frac{1}{x} + (\ln x)^p]}{(x(\ln x)^p)^2}$$

$$= \frac{-[p(\ln x)^{p-1} + (\ln x)^p]}{[x(\ln x)^p]^2} < 0$$

Because the hypotheses are met, the Integral Test applies

$$\int_2^{\infty} \frac{1}{x(\ln x)^p} dx \underset{\substack{u = \ln x \\ du = \frac{1}{x}dx}}{=} \int_{\ln 2}^{\infty} \frac{1}{u^p} du$$

From p-integrals, we know that the integral will converge if and only if $p > 1$. Therefore, $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ will converge if $p > 1$ and diverge if $p \leq 1$.

1. Decreasing	2. Other Hypoth.	3. Integral	4. Conclusion	5. Notation