10.4 Problems

8. $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\frac{1}{n}}$ By the Divergence Test, since $\lim_{n\to\infty} (-1)^n \cdot \frac{1}{1+1/n} \neq 0$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\frac{1}{n}}$ diverges.

9. $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{(\ln n)^2} = \sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$

a) Consider $\sum_{n=2}^{\infty} \left| \frac{(-1)^n}{(\ln n)^2} \right| = \sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$

Because $\ln n < n^{1/2}$ for large n, $(\ln n)^2 < n$

$$\Rightarrow \frac{1}{(\ln n)^2} > \frac{1}{n} > 0$$

Since $\sum_{n=2}^{\infty} \frac{1}{n}$ diverges, by the comp test, $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$ diverges. Thus $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$ does not converge absolutely

b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$

AST

1) $b_n = \frac{1}{(\ln n)^2} > 0$

2) $\lim_{n\to\infty} \frac{1}{(\ln n)^2} = 0$

3) $b_{n+1} = \frac{1}{(\ln(n+1))^2} < \frac{1}{(\ln n)^2} = b_n$

By the AST, $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$ converges.

Thus, $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$ converges conditionally.

10. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$

a) Consider $\displaystyle\sum_{n=1}^{\infty} \left| \frac{\cos n}{2^n} \right|$

Since $0 \le \frac{|\cos n|}{2^n} \le \frac{1}{2^n}$ and $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ converges ($|r| = 1/2 < 1$), then $\sum_{n=1}^{\infty} \left| \frac{\cos n}{2^n} \right|$ converges.
This implies that $\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$ converges absolutely.

11. $S = \displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^3}$

a) calculate S_n for $1 \le n \le 10$

$S_1 = 1$	$S_6 = 0.899782$
$S_2 = 0.875$	$S_7 = 0.902698$
$S_3 = 0.912037$	$S_8 = 0.900745$
$S_4 = 0.896412$	$S_9 = 0.902116$
$S_5 = 0.904412$	$S_{10} = 0.901116$

b) we know that $|S - S_N| \le b_{N+1}$

$$\Rightarrow |S - S_{10}| \le \frac{1}{11^3}$$

$$\Rightarrow -\frac{1}{11^3} \le S - S_{10} \le \frac{1}{11^3}$$

$$\Rightarrow S_{10} - \frac{1}{11^3} \le S \le S_{10} + \frac{1}{11^3}$$

$$\Rightarrow 0.900 \le S \le 0.902$$ ✓