Math 182 Problems Name: Key

Sections: 5.7, 5.8

Due: 16 Jan 2018 Point values in $\boxed{\text{boxes}}$.

1. $\boxed{1}$ Consider the integral $\int_{-1}^{1}(x^2+1)\,dx$. Although it is trivial to compute directly, consider the following substitution.

$$\begin{aligned}\text{Let } u &= x^2,\\ du &= 2x dx, \text{ so } \frac{du}{2\sqrt{u}} = dx,\\ x = -1 &\mapsto u = 1, \text{ and}\\ x = 1 &\mapsto u = 1.\end{aligned}$$

mistake!

Using this substitution we have

$$\int_{-1}^{1}(x^2+1)\,dx = \int_{1}^{1}\frac{u+1}{2\sqrt{u}}\,du = 0$$

which is clearly false. Where is the mistake?

if $u = x^2$, $x = \pm\sqrt{u}$ so $dx = \dfrac{du}{\pm 2\sqrt{u}}$

$$\int_{-1}^{1}(x^2+1)\,dx = \overbrace{\int_{1}^{0}\frac{u+1}{-2\sqrt{u}}\,du + \int_{0}^{1}\frac{u+1}{2\sqrt{u}}\,du}^{\text{change bounds}}$$

2. $\boxed{2}$ Integrate $\int_0^1 \frac{x+x^3}{1+x^4}\,dx$.

$$\int_0^1 \frac{x+x^3}{1+x^4}\,dx = \int_0^1 \frac{x}{1+x^4}\,dx + \underbrace{\int_0^1 \frac{x^3}{1+x^4}\,dx}_{\substack{u=1+x^4\\ du=4x^3dx}} = \underbrace{\int_0^1 \frac{x}{1+(x^2)^2}\,dx}_{\substack{u=x^2\\ du=2x\,dx}} + \frac{1}{4}\int_1^2 \frac{1}{u}\,du$$

$$= \frac{1}{2}\int_0^1 \frac{1}{1+u^2}\,du + \frac{1}{4}\left(\ln u\Big|_1^2\right)$$

$$= \frac{1}{2}\left[\arctan 1 - 0\right] + \frac{1}{4}\left[\ln 2 - \ln 1\right] = \frac{\pi}{8} + \frac{\ln 2}{4}$$

3. $\boxed{2}$ Integrate $\int \frac{dt}{\sqrt{t(1-t)}}$ using the substitution $t = \sin^2 x$ where $x \in (0, \pi/2)$.

$t = (\sin x)^2$

$dt = 2\sin x\cos x\,dx$

$$\int \frac{2\sin x\cos x}{\sqrt{\sin^2 x(1-\sin^2 x)}}\,dx = \int \frac{2\sin x\cos x}{\sqrt{\sin^2 x\cos^2 x}}\,dx$$

$+\sqrt{t} = \sin x$

$x = \arcsin(\sqrt{t})$

$$= \int 2\,dx = 2x + C$$

$$= 2\arcsin(\sqrt{t}) + C$$

4. Integrate.

(a) $\boxed{1}$ $\displaystyle\int \frac{dx}{x^2+4} = \int \frac{dx}{4(\frac{x^2}{4}+1)} = \frac{1}{4}\int \frac{dx}{(x/2)^2+1}$ $u = x/2$, $du = 1/2\, dx$

$$= \frac{2}{4}\int \frac{1}{u^2+1}\,du = \frac{1}{2}\arctan u + C$$
$$= \frac{1}{2}\arctan(x/2) + C$$

(b) $\boxed{2}$ $\displaystyle\int \frac{dx}{x^2+2x+5}$

[HINT: Complete the square.]

$x^2+2x+5 = x^2+2x+1-1+5$
$= (x+1)^2+4$

$$= \int \frac{dx}{(x+1)^2+4}$$ $u = x+1$, $du = dx$

$$= \int \frac{du}{u^2+4} = \frac{1}{2}\arctan(u/2) + C$$ (from a))

$$= \frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + C$$

(c) $\boxed{2}$ $\displaystyle\int \frac{2x+5}{x^2+2x+5}\,dx = \int \frac{2x+2+3}{x^2+2x+5}\,dx$

$$= \underbrace{\int \frac{2x+2}{x^2+2x+5}\,dx}_{\substack{u = x^2+2x+5 \\ du = (2x+2)dx}} + \int \frac{3}{x^2+2x+5}\,dx$$

part b)

$$= \int \frac{1}{u}\,du + 3\int \frac{1}{x^2+2x+5}\,dx$$

$$= \ln|x^2+2x+5| + \frac{3}{2}\arctan\left(\frac{x+1}{2}\right) + C$$