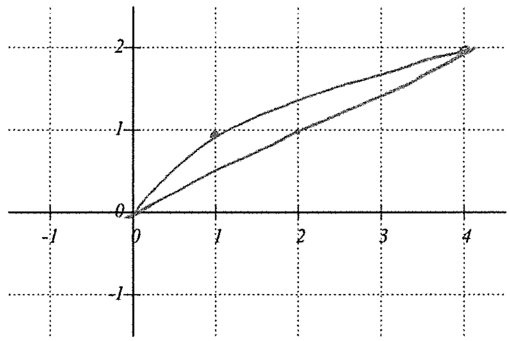
Math 182 Problems **Name:** Key

Sections: 6.3, 6.4 Part 1

Due: 26 Jan 2018 Point values in $\boxed{\text{boxes}}$.

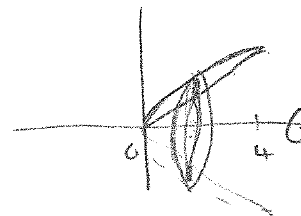
1. Carefully sketch the region bounded by the graphs of $y = \sqrt{x}$ and $y = x/2$.



$y = \sqrt{x} \Leftrightarrow y^2 = x$

$y = \frac{x}{2} \Leftrightarrow 2y = x$

(a) $\boxed{2}$ The region is rotated about the x-axis. Express the volume using both the Disk Method and the Shell Method. Do no integrate.

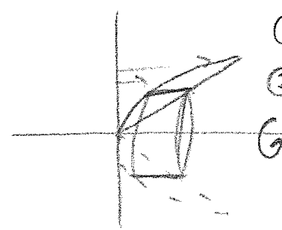


① var: x

② $r_o = \sqrt{x}$

$r_i = \frac{x}{2}$

$$V_{Disk} = \int_0^4 \pi\left((\sqrt{x})^2 - (x/2)^2\right)\,dx$$



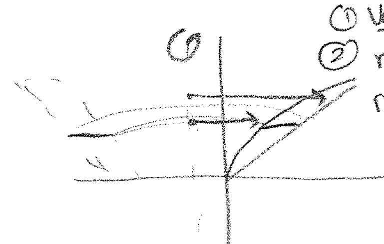
① var: y

② $r = y$

$h = 2y - y^2$

$$V_{Shell} = \int_0^2 2\pi y(2y - y^2)\,dy$$

(b) $\boxed{2}$ The region is rotated about the line $x = -1$. Express the volume using both the Disk Method and the Shell Method. Do no integrate.

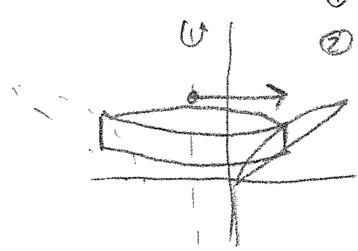


① var: y

② $r_o = 1 + 2y$

$r_i = 1 + y^2$

$$V_{Disk} = \int_0^2 \pi\left((1+2y)^2 - (1+y^2)^2\right)\,dy$$



① var: x

② $r = x + 1$

$h = \sqrt{x} - \frac{x}{2}$

$$V_{Shell} = \int_0^4 2\pi(x+1)\left(\sqrt{x} - \frac{x}{2}\right)\,dx$$

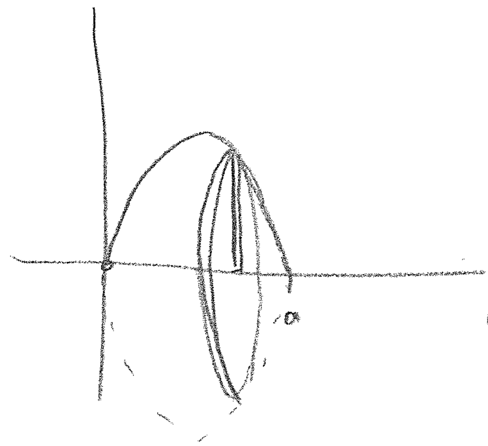
2. $\boxed{3}$ For $a > 0$, consider the region bounded by the the graph of $y = ax - x^2$ the x-axis. The region is rotated about the x-axis. Compute the volume.

$y = x(a-x)$

$y = x(a-x)$

$x(a-x) = 0$

$x = 0, \quad x = a$



disks

① variable : x

② $r = y = ax - x^2$

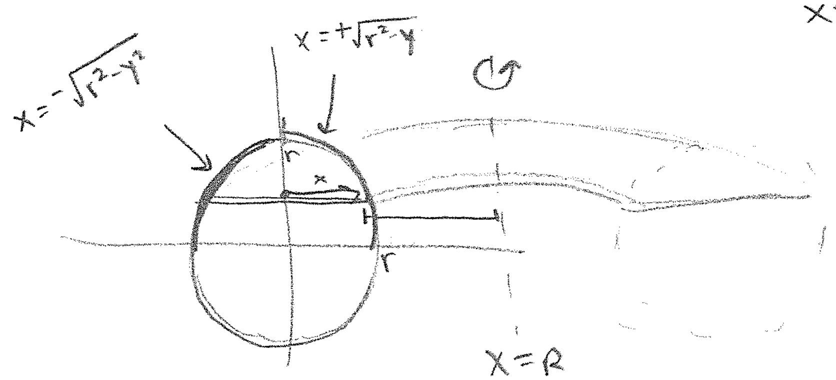
③ $V = \int_0^a \pi (ax - x^2)^2 \, dx$

$= \int_0^a \pi \left[a^2x^2 - 2ax^3 + x^4 \right] dx$

$= \pi \left[\frac{a^2x^3}{3} - \frac{2ax^4}{4} + \frac{x^5}{5} \Big|_0^a \right] = \pi \left[\frac{a^5}{3} - \frac{a^5}{2} + \frac{a^5}{5} \right] = \frac{\pi a^5}{30}$

3. $\boxed{3}$ A torus is generated by rotating a circle of radius r about a line R units from the center of the circle. Use the Disk Method to compute the volume.

$x^2 + y^2 = r^2$



① variable : y

② $r_o = R - x = R - (-\sqrt{r^2-y^2})$

$r_i = R - x = R - (\sqrt{r^2-y^2})$

$V = 2\int_0^r \pi \left[(R + \sqrt{r^2-y^2})^2 - (R - \sqrt{r^2-y^2})^2 \right]$

$V = 2\pi \int_0^r \left[R^2 + 2R\sqrt{r^2-y^2} + (r^2-y^2) - \left(R^2 - 2R\sqrt{r^2-y^2} + (r^2-y^2) \right) \right] dy$

$= 2\pi \int_0^r 4R\sqrt{r^2-y^2} \, dy = 8\pi R \underbrace{\int_0^r \sqrt{r^2-y^2} \, dy}_{\text{area of } 1/4 \text{ a circle}} = 8\pi R \frac{1}{4} \pi r^2$

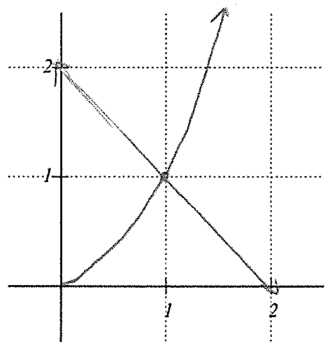
$= (2\pi R)(\pi r^2)$

Math 182 Problems **Name:** Key

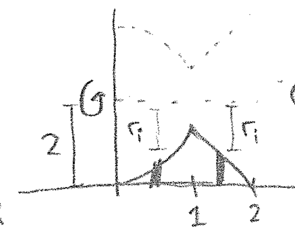
Sections: 6.3, 6.4 Part 2

Due: 26 Jan 2018 Point values in $\boxed{\text{boxes}}$.

1. Carefully sketch the region bounded by the graphs of $y = x^3$ and $y = 2 - x$ and the x-axis.



(a) $\boxed{2}$ The region is rotated about the line $y = 2$. Express the volume using both the Disk Method and the Shell Method. Do not integrate.



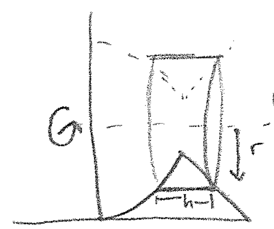
① var: x

② $r_{outer} = 2$

$r_{inner} =$ DEPENDS!

∗ 2 integrals

$$V_{Disk} = \int_0^1 \pi\left(2^2 - (2-x^3)^2\right)dx + \int_1^2 \pi\left(2^2 - (x^2)\right)dx$$



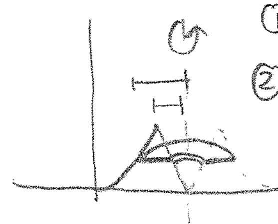
① var: y

② $r = 2-y$

$h = 2-y-y^{1/3}$

$$V_{Shell} = \int_0^1 2\pi(2-y)(2-y-y^{1/3})\,dy$$

(b) $\boxed{2}$ The region is rotated about the line $x = 2$. Express the volume using both the Disk Method and the Shell Method. Do not integrate.



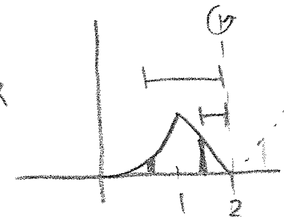
① var: y

② $r_i = 2-(2-y)$

$= y$

$r_o = 2 - y^{1/3}$

$$V_{Disk} = \int_0^1 \pi\left((2-y^{1/3})^2 - y^2\right)dy$$



① var: x

② $r = 2-x$

$h =$ DEPENDS!

∗ 2 integrals

$$V_{Shell} = \int_0^1 2\pi(2-x)(x^3)\,dx + \int_1^2 2\pi(2-x)(2-x)\,dx$$

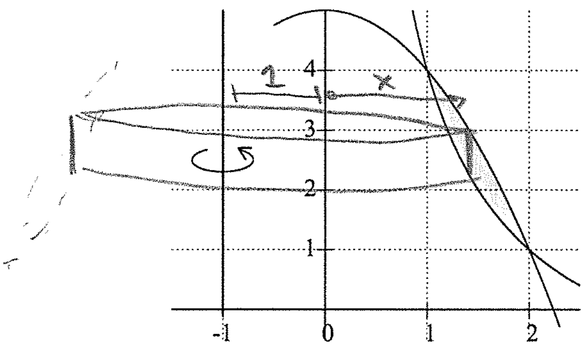
2. $\boxed{3}$ The region in the first quadrant bounded by the graphs of $y = 5 - x^2$ and $y = 6/x - 2$, (the shaded region in the figure) is rotated about the line $x = -1$. Compute the volume.

Shells

① variable: x

② $r = (1+x)$

$h = 5 - x^2 - (6/x - 2) = 7 - x^2 - \frac{6}{x}$



$$V = \int_1^2 2\pi(1+x)\left(7 - x^2 - \frac{6}{x}\right) dx$$

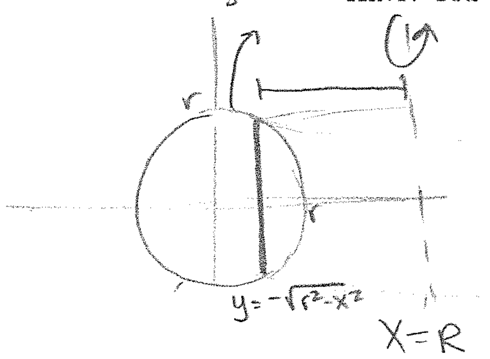
$$= 2\pi \int_1^2 7 - x^2 - \frac{6}{x} + 7x - x^3 - 6 \, dx = 2\pi \int_1^2 1 - x^3 - x^2 + 7x - \frac{6}{x} \, dx$$

$$= 2\pi \left[\left(x - \frac{x^4}{4} - \frac{x^3}{3} + \frac{7x^2}{2} - 6\ln x \right) \Big|_1^2 \right] = 2\pi\left[2 - 4 - \frac{8}{3} + 14 - 6\ln 2 - \left(1 - \frac{1}{4} - \frac{1}{3} + \frac{7}{2}\right)\right]$$

$$= 2\pi\left[\frac{28}{3} - 6\ln 2 - \frac{47}{12}\right]$$

3. $\boxed{3}$ A torus is generated by rotating a circle of radius r about a line R units from the center of the circle. Use the Shell Method to compute the volume.

HINT: You will find it useful to interpret the integral $\int_0^r \sqrt{r^2 - t^2}\, dt$ as an area.



① var: x

② $r = R - x$

$h = \sqrt{r^2-x^2} - -\sqrt{r^2-x^2} = 2\sqrt{r^2-x^2}$

$$V = \int_{-r}^{r} 2\pi(R-x)2\sqrt{r^2-x^2}\, dx$$

$$= 4\pi \int_{-r}^{r} R\sqrt{r^2-x^2} - x\sqrt{r^2-x^2}\, dx = 4\pi\left[\underbrace{R\int_{-r}^{r}\sqrt{r^2-x^2}\,dx}_{\frac{1}{2}\text{ of a circle}} - \int_{-r}^{r} x\sqrt{r^2-x^2}\,dx\right]$$

$u = r^2 - x^2$

$du = -2x\,dx$

$$= 4\pi\left[\frac{R\pi r^2}{2} + \underbrace{\frac{1}{2}\int_0^0 \sqrt{u}\,du}_{0}\right] = (2\pi R)(\pi r^2)$$