Math 182 Problems **Name:** Key

Sections: 7.1 Point values in $\boxed{\text{boxes}}$.

Due: 6 Feb 2018

1. $\boxed{2}$ In class we showed

$$\int \ln x\, dx = x \ln x - x + c$$

using integration by parts by letting $u = \ln x, dv = dx$.
Logs can also be integrated with an appropriate substitution. Evaluate the same integral by starting with the substitution $x = e^u$, and then use integration by parts on the resulting integral. Verify that your solution is consistent with what we derived in class.

$x = e^u \Leftrightarrow \ln x = u$

$dx = e^u du$

$$\int \ln e^u \cdot e^u\, du = \int u e^u\, du$$

IBP

$w = u \quad dv = e^u du$

$dw = du \quad v = e^u$

$$= ue^u - \int e^u du = ue^u - e^u + c$$

$$= \ln x\, e^{\ln x} - e^{\ln x} + c = x \ln x - x + c$$

2. $\boxed{3}$ In class we discussed the LIATE rule of thumb for integration by parts.

(a) Show that the rule fails spectacularly for

$$\int \frac{xe^x}{(x+1)^2}\, dx.$$

Specifically, choose the Algebraic expression for u, i.e. $u = \frac{x}{(x+1)^2}$, and the Exponential expression for dv, i.e. $dv = e^x\, dx$. Perform the integration by parts and notice that the integral is much worse than you started with.

IBP

$u = \frac{x}{(x+1)^2} \quad dv = e^x dx$

$du = \frac{(x+1)^2 - x(2(x+1))}{(x+1)^4} dx \quad v = e^x$

$= \frac{(1-x)}{(x+1)^3} dx$

$$\int \frac{xe^x}{(x+1)^2} dx = \frac{xe^x}{(x+1)^2} - \underbrace{\int \frac{(1-x)e^x}{(x+1)^3} dx}_{\text{more difficult than original integral}}$$

(b) Use an appropriate choice for u and dv to perform the integration using integration by parts.
[HINT: Use some scratch paper.]

$u = xe^x \quad dv = \frac{1}{(x+1)^2} dx$

$du = (xe^x + e^x) dx \quad v = \frac{-1}{(x+1)}$

factor out e^x

$$\int \frac{xe^x}{(x+1)^2} dx = \frac{-xe^x}{(x+1)} - -\int \frac{xe^x + e^x}{(x+1)} dx = \frac{-xe^x}{(x+1)} + \int \frac{\cancel{(x+1)}e^x}{\cancel{(x+1)}} dx$$

$$= \frac{-xe^x}{x+1} + \int e^x dx = \frac{-xe^x}{x+1} + e^x + c$$

3. $\boxed{1}$ Consider

$$\int \frac{dx}{x}. \tag{1}$$

Applying integration by parts with

$$u = \frac{1}{x} \qquad dv = dx \tag{2}$$

$$du = \frac{-1}{x^2}\,dx \qquad v = x \tag{3}$$

gives

$$\int \frac{dx}{x} = 1 + \int \frac{dx}{x}. \tag{4}$$

Subtracting the integral from both sides gives

$$0 = 1. \tag{5}$$

Explain the error in the above.

weve lost a constant $(+C)$!

consider: $\int \frac{dx}{x} = 1 + \int \frac{dx}{x}$

$\ln|x| + C_1 = 1 + \ln|x| + C_2 \Rightarrow C_1 = 1 + C_2$

4. $\boxed{4}$ Verify the reduction identity

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx.$$

IBP

$u = \cos^{n-1} x \qquad dv = \cos x\,dx$

$du = (n-1)\cos^{n-2} x\,(-\sin x)dx \qquad v = \sin x$

$$\int \cos^{n-1} x \cos x\,dx = \sin x \cos^{n-1} x - -\int (n-1)\cos^{n-2} x \underbrace{\sin^2 x}_{1-\cos^2 x}\,dx$$

$$= \sin x \cos^{n-1} x + (n-1)\int \cos^{n-2} x - \cos^n x\,dx$$

so,

$$\int \cos^n x\,dx = \sin x \cos^{n-1} x + (n-1)\int \cos^{n-2} x\,dx \underbrace{- (n-1)\int \cos^n x\,dx}_{\text{add over}}$$

$$n\int \cos^n x\,dx = \sin x \cos^{n-1} x + (n-1)\int \cos^{n-2} x\,dx$$

$$\int \cos^n x\,dx = \frac{1}{n}\sin x \cos^{n-1} x + \frac{n-1}{n}\int \cos^{n-2} x\,dx \quad ✓$$