Math 182 Problems **Name:** Key

Sections: 7.2 Point values in $\boxed{\text{boxes}}$.

Due: 9 Feb 2018

1. $\boxed{5}$ Use these identities to evaluate the following integrals. Assume $m, n \in \mathbb{N}$.

$$\begin{aligned}
\sin A \ \cos B &= \frac{1}{2}(\sin(A+B) + \sin(A-B)) \\
\cos A \ \cos B &= \frac{1}{2}(\cos(A+B) + \cos(A-B)) \\
\sin A \ \sin B &= \frac{1}{2}(\cos(A-B) - \cos(A+B))
\end{aligned}$$

(a) $\int_{-\pi}^{\pi} \cos(nx)\cos(mx)\,dx$ for $m \neq n$

$$= \int_{-\pi}^{\pi} \frac{1}{2}\left[\cos((n+m)x) + \cos((n-m)x)\right] dx = \frac{1}{2}\left[\frac{\sin((n+m)x)}{n+m} + \frac{\sin((n-m)x)}{n-m}\right]\Bigg|_{-\pi}^{\pi}$$

$$= \frac{1}{2}\left[\underbrace{\frac{\sin((n+m)\pi)}{n+m}}_{0} + \underbrace{\frac{\sin((n-m)\pi)}{n-m}}_{0} - \left(\underbrace{\frac{\sin((n+m)(-\pi))}{n+m}}_{0} + \underbrace{\frac{\sin((n-m)(-\pi))}{n-m}}_{0}\right)\right] = \boxed{0}$$

(b) $\int_{-\pi}^{\pi} \cos(nx)\cos(mx)\,dx$ for $m = n$

$$= \int_{-\pi}^{\pi} \cos^2(nx)\,dx = \int_{-\pi}^{\pi} \frac{1}{2}\left[1 + \cos(2nx)\right] dx$$

$$= \frac{1}{2}\left[x + \frac{\sin(2nx)}{2n}\right]\Bigg|_{-\pi}^{\pi} = \frac{1}{2}\left[\pi - (-\pi)\right] = \boxed{\pi}$$

(c) $\int_{-\pi}^{\pi} \sin(nx)\cos(mx)\,dx$

$$= \int_{-\pi}^{\pi} \frac{1}{2}\left[\sin((n+m)x) + \sin((n-m)x)\right] dx = \frac{1}{2}\left[-\frac{\cos((n+m)x)}{n+m} - \frac{\cos((n-m)x)}{n-m}\right]\Bigg|_{-\pi}^{\pi}$$

$$= \frac{1}{2}\left[-\frac{\cos((n+m)\pi)}{n+m} - \frac{\cos((n-m)\pi)}{n-m} - \left(\underbrace{-\frac{\cos((n+m)(-\pi))}{n+m}}_{= -\frac{\cos((n+m)\pi)}{n+m}} - \underbrace{\frac{\cos((n-m)(-\pi))}{n-m}}_{= -\frac{\cos((n-m)(+\pi))}{n-m}}\right)\right]$$

b/c cosine is an even function: $f(-x) = f(x)$

$$= \frac{1}{2}[0] = \boxed{0}$$

2. $\boxed{5}$ Evaluate.

(a) $\frac{1}{\pi}\int_0^{\pi} x\cos(nx)\,dx = \frac{1}{\pi}\left[\frac{x\sin(nx)}{n}\Big|_0^{\pi} - \int_0^{\pi} \frac{\sin(nx)}{n}\,dx \right]$

IBP $\quad u = x \quad dv = \cos(nx)\,dx$
$\quad du = dx \quad v = \frac{\sin(nx)}{n}$

$= \frac{1}{\pi}\left[0 - -\frac{\cos(nx)}{n^2}\Big|_0^{\pi} \right]$

$= \frac{1}{\pi n^2}\left[\cos(n\pi) - 1\right] = \boxed{\begin{cases} \frac{-2}{\pi n^2} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}}$

(b) $\int \cot^5(4x)\csc^3(4x)\,dx = \int \cot^4(4x)\csc^2(4x)\underbrace{\csc(4x)\cot(4x)\,dx}_{\text{"}du\text{"}}$

$u = \csc(4x)$

$du = -4\csc(4x)\cot(4x)\,dx$

$\rightarrow \frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$1 + \cot^2\theta = \csc^2\theta$

$= \int (\cot^2(4x))^2 \csc^2(4x)\,\underline{\csc(4x)\cot(4x)\,dx}$

$= \int (\csc^2(4x) - 1)^2 \csc^2(4x)\,\underline{\csc(4x)\cot(4x)\,dx}$

$= -\frac{1}{4}\int (u^2-1)^2 u^2\,du = -\frac{1}{4}\int u^6 - 2u^4 + u^2\,du$

$\boxed{= -\frac{1}{4}\left[\frac{\csc^7(4x)}{7} - 2\frac{\csc^5(4x)}{5} + \frac{\csc^3(4x)}{3} \right] + C}$

(c) $\int_{\pi/6}^{\pi/2} \cot^2 x\,dx$

$= \int_{\pi/6}^{\pi/2} \csc^2 x - 1\,dx = (-\cot x - x)\Big|_{\pi/6}^{\pi/2}$

$\cot\frac{\pi}{2} = \frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}} = \frac{0}{1}$

$\cot\frac{\pi}{6} = \frac{\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}} = \frac{\sqrt{3}/2}{1/2}$

$= -\cot\frac{\pi}{2} - \frac{\pi}{2} - \left(-\cot\frac{\pi}{6} - \frac{\pi}{6}\right)$

$= -\frac{\pi}{2} + \sqrt{3} + \frac{\pi}{6}$

$= \sqrt{3} - \frac{\pi}{3}$