

Submit your neatly written solutions, in order, on paper stapled to this sheet.

1. 4 Let

$$f(x) = \frac{2x^3 + 9x^2 + 19x + 20}{x^3 + 3x^2 + 7x + 5}.$$

Evaluate  $\int f(x) dx$ .

[HINT 1:  $f(x)$  is not a proper rational expression, reduce it to a proper rational expression.]

[HINT 2:  $x^3 + 3x^2 + 7x + 5 = (x + 1)(x^2 + 2x + 5)$ ]

2. 3 Consider the integral

$$\int \frac{dt}{1 + e^{2t}}. \tag{1}$$

- (a) The brute force method to evaluate (1) is to use the substitution  $u = 1 + e^{2t}$ . Show that using this substitution converts the integral into

$$\int \frac{dt}{1 + e^{2t}} = \int \frac{du}{2u(u - 1)}$$

and then evaluate the integral using partial fraction decomposition.

- (b) A few years ago Adrian Soto was a grad student at MSU. He pointed out a trick for dealing with integrals like (1). Adrian’s trick is to multiply the integrand by  $e^{-2t}/e^{-2t}$  and then perform the obvious substitution. Evaluate the integral using Adrian’s trick.

- (c) Show your solutions to (a) and (b) are equivalent.

3. 2 When we discussed  $\int \sec x dx$ , a random “let’s multiply the integrand by  $(\sec x + \tan x)/(\sec x + \tan x)$ ” was pulled out of thin air<sup>1</sup>. Although it is a useful trick, let’s approach  $\int \sec x dx$  in a slightly more “follow your nose” approach,

$$\int \sec x dx = \int \frac{1}{\cos x} dx = \int \frac{\cos x}{\cos^2 x} dx = \int \frac{\cos x}{1 - \sin^2 x} dx.$$

Use the substitution  $u = \sin x$  on the final integral and then partial fractions to derive

$$\int \sec x dx = \ln |\sec x + \tan x| + c \tag{2}$$

directly, instead of using the trick that we used in class. Note, there is a fair amount of algebra and trigonometry to do after the integration to get it in the form of (2).

4. 1 Submit your neatly written solutions, in order, on paper stapled to this sheet. Do not submit scratch paper<sup>2</sup>.

<sup>1</sup>By the time you finish this you hopefully will gain an appreciation for the trick!

<sup>2</sup>Full solutions are expected, i.e. please show all necessary steps. However, do not include work that you know is incorrect, nor work that ends up not being relevant to your final solutions.