# 8.1 | 8.2 problems

1. Find the length of $y = \int_1^x \sqrt{\sqrt{t}-1}\,dt$ for $x \in [0,16]$.

1) $y' = \sqrt{\sqrt{x}-1}$ by FTC

2) $(y')^2 = \sqrt{x} - 1$

3) $(y')^2 + 1 = \sqrt{x}$

$$S = \int_0^{16} \sqrt{\sqrt{x}}\,dx = \int_0^{16} x^{1/4}\,dx$$

$$= \frac{4}{5}\left[ x^{5/4} \Big|_0^{16} \right] = \frac{128}{5}$$

2. Find length of $y = \sqrt{x - x^2} + \arcsin(\sqrt{x})$.

The curve is valid for $x - x^2 \geq 0$

$\Rightarrow x(1-x) \geq 0$

$\Rightarrow x \geq 0$ and $x \leq 1$ , $[0,1]$

1) $y' = \dfrac{1-2x}{2\sqrt{x-x^2}} + \dfrac{1}{\sqrt{1-x}} \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{1-2x+1}{2\sqrt{x-x^2}} = \dfrac{1-x}{\sqrt{x-x^2}}$

same denom.

2) $(y')^2 = \dfrac{(1-x)^2}{x(1-x)} = \dfrac{1-x}{x} = \dfrac{1}{x} - 1$

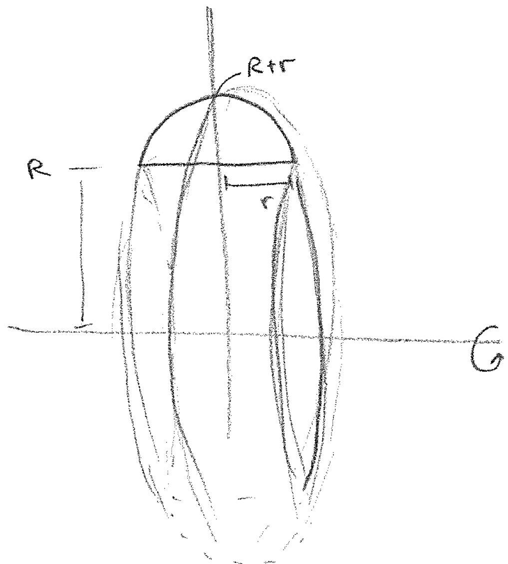
3) $(y')^2 + 1 = \dfrac{1}{x}$

$$S = \int_0^1 \sqrt{\frac{1}{x}}\,dx = \lim_{R \to 0^+} \int_R^1 x^{-1/2}\,dx = \lim_{R \to 0^+} 2\sqrt{x} \Big|_R^1 = \lim_{R \to 0^+} 2 - 2\sqrt{R}$$

$$= 2$$

3. Find the surface area obtained by rotating

$y = R + \sqrt{r^2 - x^2}$ $(R > r > 0)$ about $x$-axis.




$$SA = \int_{-r}^{r} 2\pi \left(R + \sqrt{r^2 - x^2}\right)\left(\sqrt{\frac{r^2}{r^2 - x^2}}\right) dx$$

↑ from below

$$= 4\pi \int_0^r \left(R + \sqrt{r^2 - x^2}\right) \frac{r}{\sqrt{r^2 - x^2}} \, dx$$

by symmetry

$$= 4\pi r \left[ \int_0^r \frac{R}{\sqrt{r^2 - x^2}} \, dx + \int_0^r 1 \, dx \right]$$

$$= 4\pi r \left[ R \cdot \lim_{S \to r^-} \int_0^S \frac{1}{\sqrt{r^2 - x^2}} \, dx + r \right]$$

$$= 4\pi r \left[ R \cdot \left( \lim_{S \to r^-} \arcsin\left(\frac{S}{r}\right) - 0 \right) + r \right]$$

$$= 4\pi r \left[ R \cdot \arcsin 1 + r \right]$$

Aside

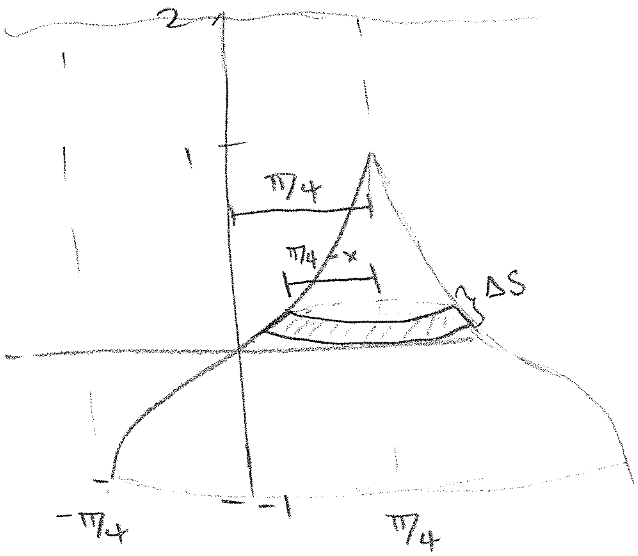
1) $y' = \dfrac{-2x}{2\sqrt{r^2 - x^2}} = \dfrac{-x}{\sqrt{r^2 - x^2}}$

2) $(y')^2 = \dfrac{x^2}{r^2 - x^2}$

3) $(y')^2 + 1 = \dfrac{x^2}{r^2 - x^2} + 1 = \dfrac{r^2}{\sqrt{r^2 - x^2}}$

$$= 4\pi r \left[ \frac{R\pi}{2} + r \right]$$

4. a) Find fluid force acting on top of solid.




$y = \tan x$

$x = \arctan y$

$\Delta S = \sqrt{1 + (f'(y))^2}\,\Delta y$

$= \sqrt{1 + \left(\frac{1}{1+y^2}\right)^2}\,\Delta y$

1) $A_{slice} = l \cdot w = 2\pi(\pi/4 - x)\,\Delta S = 2\pi(\pi/4 - \arctan y)\sqrt{1 + \frac{1}{(1+y^2)^2}}\,\Delta y$

2) $P_{slice} = \rho g \text{ depth} = \rho g(2-y)$

$$F = \int_{-1}^{1} 2\pi\rho g\,(\pi/4 - \arctan y)\sqrt{1 + \left(\frac{1}{1+y^2}\right)^2}\,(2-y)\,dy$$

$= \underline{401374}$ N