Math 182 Quiz 1 **Name:** Key

19 Jan 2018 Point values in $\boxed{\text{boxes}}$.

1. $\boxed{3}$ Integrate.

$$\int_0^{\sqrt{3}} \left(2 + t^3\sqrt{t^2+1}\right)\,dt = \int_0^{\sqrt{3}} 2\,dt + \int_0^{\sqrt{3}} \underbrace{t^3\sqrt{t^2+1}\,dt}$$

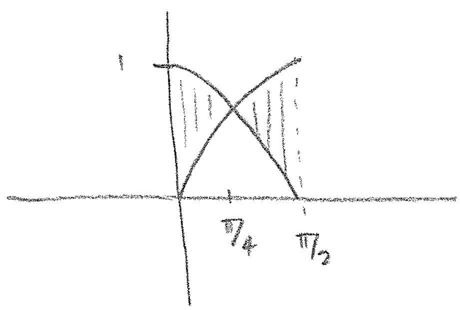
$u = t^2+1$
$du = 2t\,dt$
$t^2 = u-1$

$$= 2t\Big|_0^{\sqrt{3}} + \int_1^4 \frac{1}{2}(u-1)\sqrt{u}\,du$$

$$= 2\sqrt{3} + \frac{1}{2}\int_1^4 u^{3/2} - u^{1/2}\,du = 2\sqrt{3} + \frac{1}{2}\left[\frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3}\Bigg|_1^4\right]$$

$$= 2\sqrt{3} + \left[\frac{4^{5/2}}{5} - \frac{4^{3/2}}{3} - \frac{1}{5} + \frac{1}{3}\right] = 2\sqrt{3} + \frac{31}{5} - \frac{7}{3}$$

2. $\boxed{3}$ Carefully sketch the region between the graphs of $y = \cos x$ and $y = \sin x$ for $x \in [0, \pi/2]$. Find the area of the region.

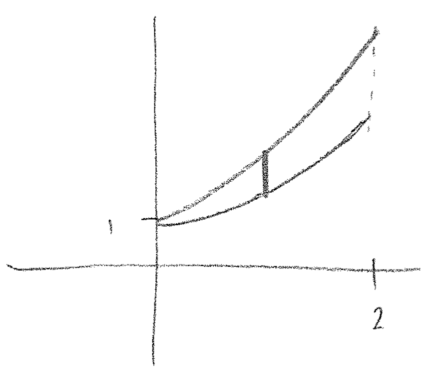


$$\int_0^{\pi/4} \cos x - \sin x\,dx + \int_{\pi/4}^{\pi/2} \sin x - \cos x\,dx$$

$$= 2\int_0^{\pi/4} \cos x - \sin x\,dx = 2\left[\sin x + \cos x\Big|_0^{\pi/4}\right]$$

$$= 2\left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0+1)\right] = 2\sqrt{2} - 2$$

3. $\boxed{4}$ A solid has base bounded between the graphs of $y = e^{2x}$ and $y = e^x$ for $x \in [0, 2]$. Cross sections prependicular to the x-axis are squares. Find the volume of the solid.



variable: x

$A_{\text{slice}} = (\text{side})^2 = (e^{2x} - e^x)^2$

$$V = \int_0^2 \underbrace{(e^{2x} - e^x)^2}_{\text{FOIL}}\,dx = \int_0^2 e^{4x} - 2e^{3x} + e^{2x}\,dx$$

$$= \frac{e^{4x}}{4} - \frac{2e^{3x}}{3} + \frac{e^{2x}}{2}\Bigg|_0^2$$

$$= \frac{e^8}{4} - \frac{2e^6}{3} + \frac{e^4}{2} - \left(\frac{1}{4} - \frac{2}{3} + \frac{1}{2}\right)$$