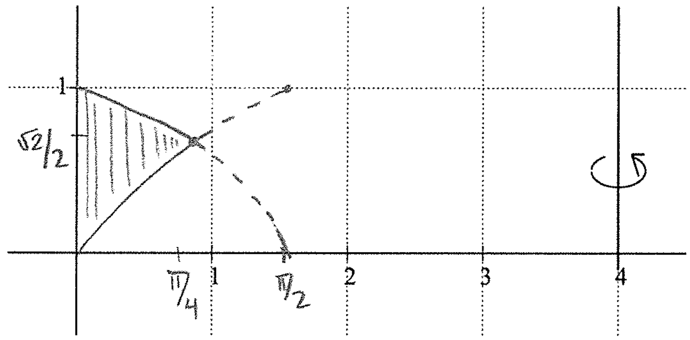
Math 182 Quiz 2 **Name:** key

26 Jan 2018 Point values in $\boxed{\text{boxes}}$.

1. $\boxed{5}$ Carefully sketch the region bounded by the graphs of $y = \sin x$ and $y = \cos x$ for $x \in \left[0, \frac{\pi}{4}\right]$. This region is rotated about the line $x = 4$. Express the volume of the resulting solid as an integral using both the Disk Method and the Shell Method. Do not integrate.



$y = \sin x \Leftrightarrow x = \arcsin y$

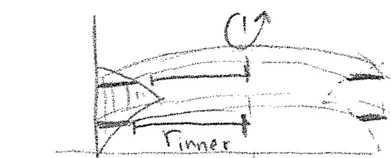
$y = \cos x \Leftrightarrow x = \arccos y$

intersect

$\sin x = \cos x$

$x = \pi/4$

DISKS/WASHERS



① variable : y

② * 2 integrals *

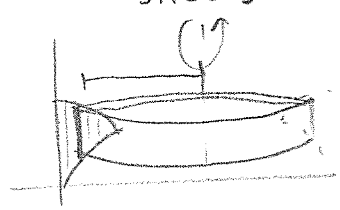
1. $r_{outer} = 4$
   $r_{inner} = 4 - \arcsin y$

2. $r_{outer} = 4$
   $r_{inner} = 4 - \arccos y$

(a) $V_{Disk} = \displaystyle\int_0^{\sqrt{2}/2} \pi\left(4^2 - (4 - \arcsin y)^2\right) dy + \int_{\sqrt{2}/2}^{1} \pi\left(4^2 - (4 - \arccos y)^2\right) dy$

(b) $V_{Shell} = \displaystyle\int_0^{\pi/4} 2\pi(4-x)(\cos x - \sin x)\, dx$

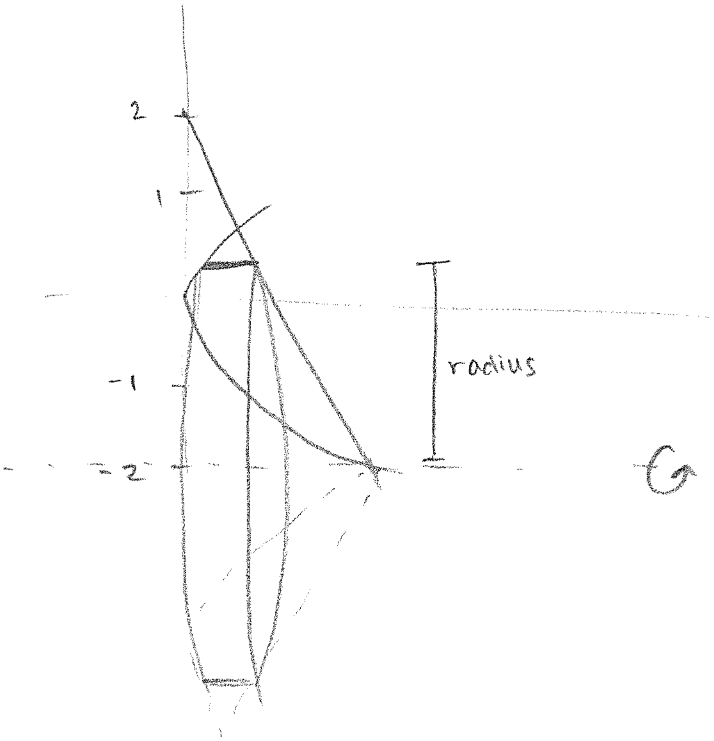
SHELLS



① variable : x

② $r = 4 - x$
$h = \cos x - \sin x$

2. $\boxed{5}$ Find the volume of the solid generated by rotating the region bounded by the graphs of $x = y^2$ and $x = 2 - y$ about the line $y = -2$.



intersect

$y^2 = 2 - y$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = -2,\ y = 1$

disks/washers $\rightarrow$ 2 integrals

Shells $\rightarrow$ 1 integral

Shells

① variable: y

② $r = y + 2$

$h = (2-y) - y^2$

$$V = \int_{-2}^{1} 2\pi (y+2)(2-y-y^2)\,dy$$

$$= 2\pi \int_{-2}^{1} 2y - y^2 - y^3 + 4 - 2y - 2y^2\,dy = 2\pi \int_{-2}^{1} -y^3 - 3y^2 + 4\,dy$$

$$= 2\pi \left[-\frac{y^4}{4} - y^3 + 4y \Big|_{-2}^{1} \right] = 2\pi \left[-\frac{1}{4} - 1 + 4 - (-4 + 8 - 8) \right]$$

$$= \frac{27\pi}{2}$$

Disks: $V = \int_0^1 \pi \left[(\sqrt{x}+2)^2 - (-\sqrt{x}+2)^2 \right] dx + \int_1^4 \pi \left[(4-x)^2 - (-\sqrt{x}+2)^2 \right] dx$