Math 182 Quiz **Name:** Key

9 Feb 2018 Point values in $\boxed{\text{boxes}}$.

Given: $\sin(2x) = 2\sin x\cos x$ || $\sin^2 x = (1-\cos(2x))/2$ || $\cos^2 x = (1+\cos(2x))/2$ || $\int \sec x = \ln|\sec x + \tan x| + C$

1. Integrate.

(a) $\boxed{3}$ $\displaystyle\int_1^e x^2 \ln x\,dx$

$u = \ln x \qquad dv = x^2\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{x^3}{3}$

$$= \frac{x^3}{3}\ln x\Big|_1^e - \int_1^e \frac{x^2}{3}\,dx$$

$$= \frac{e^3}{3}(1) - 0 - \left(\frac{x^3}{9}\Big|_1^e\right)$$

$$\boxed{= \frac{e^3}{3} - \frac{e^3}{9} + \frac{1}{9}}$$

(b) $\boxed{2}$ $\displaystyle\int \sec x\tan^5 x\,dx = \int \tan^4 x\,\underbrace{\sec x\tan x\,dx}_{du}$

$u = \sec x$

$du = \sec x\tan x\,dx$

$$= \int (\tan^2 x)^2 \sec x\tan x\,dx$$

$$= \int (\sec^2 x - 1)^2 \sec x\tan x\,dx$$

$$= \int (u^2-1)^2\,du = \int (u^4 - 2u^2 + 1)\,du$$

$$= \frac{u^5}{5} - \frac{2u^3}{3} + u + C$$

$$\boxed{= \frac{\sec^5 x}{5} - \frac{2\sec^3 x}{3} + \sec x + C}$$

Continued on the reverse.

(c) $\boxed{3}$ Evaluate

$$\int e^{\sin x} \sin(2x)\, dx.$$

[HINT: Turn your paper over.]

$$\sin 2x = 2\sin x \cos x$$

$$= \int e^{\sin x} 2\sin x \cos x \, dx \qquad w = \sin x \quad dw = \cos x\, dx$$

$$= \int e^{w} 2w\, dw \qquad \text{IBP} \quad u = 2w \quad dv = e^{w} dw \quad du = 2dw \quad v = e^{w}$$

$$= 2we^{w} - \int 2e^{w} dw$$

$$= 2we^{w} - 2e^{w} + C = \boxed{2\sin x e^{\sin x} - 2e^{\sin x} + C}$$

(d) $\boxed{2}$ Consider the integral

$$\int \sqrt{16 - t^2}\, dt.$$

Applying the trigonometric substitution

$$t = 4\sin\theta$$

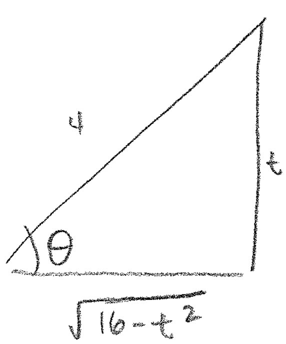
converts the integral into

$$\int 16\cos^2\theta\, d\theta.$$

Evaluating the integral gives

$$8\theta + 4\sin 2\theta + C. \tag{1}$$

Use an appropriate triangle to convert (1) back to the original variables.



$$t = 4\sin\theta$$

$$\frac{t}{4} = \sin\theta \iff \arcsin(t/4) = \theta$$

$$8\theta + 4\sin 2\theta + C = 8\theta + 4 \cdot 2\sin\theta\cos\theta + C$$

identity

$$= 8\arcsin(t/4) + 8\,\frac{t}{4} \cdot \frac{\sqrt{16-t^2}}{4} + C$$

$$\boxed{= 8\arcsin(t/4) + \frac{t\sqrt{16-t^2}}{2} + C}$$