Math 182 Quiz **Name:** Key

15 Feb 2018

Point values in $\boxed{\text{boxes}}$.

Given: $\sin(2x) = 2\sin x\cos x$ || $\sin^2 x = (1-\cos(2x))/2$ || $\cos^2 x = (1+\cos(2x))/2$ || $\int \csc x = \ln|\csc x - \cot x| + C$

1. $\boxed{2}$ Answer the following questions:

   (a) Write out the partial fraction decomposition for the following proper rational expression. Do NOT find coefficients.

   $$\frac{2x^2+5x+11}{(x^2-1)(2x^2+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{Dx+E}{2x^2+1}$$

   $(x^2-1) \downarrow (x+1)(x-1)$

   (b) State the appropriate trigonometric substitution for the following integral: $\int \frac{1}{x^3\sqrt{4x^2-9}}\,dx$.

   $2x = 3\sec\theta$

2. $\boxed{4}$ Evaluate

$$\int \frac{1}{\sqrt{x}(x-a^2)}\,dx.$$

[Hint: Start with a substitution.]

$u = \sqrt{x} \iff u^2 = x$

$du = \frac{1}{2\sqrt{x}}\,dx$

$= \int \frac{2}{u^2-a^2}\,du$

$= \int \frac{2}{(u-a)(u+a)}\,du$

$\frac{2}{(u-a)(u+a)} = \frac{A}{u-a} + \frac{B}{u+a}$

$2 = A(u+a) + B(u-a)$

let $u = a$: $2 = A(2a)$

$A = 1/a$

let $u=-a$: $2 = B(-2a)$

$B = -1/a$

$= \int \frac{1/a}{u-a} + \frac{-1/a}{u+a}\,du$

$= \frac{1}{a}\ln|u-a| - \frac{1}{a}\ln|u+a| + C$

$= \frac{1}{a}\ln|\sqrt{x}-a| - \frac{1}{a}\ln|\sqrt{x}+a| + C$

Continued on the reverse.

3. $\boxed{4}$ Evaluate

$$\int \frac{dx}{(x^2+16)^2}.$$

$x = 4\tan\theta$

$dx = 4\sec^2\theta \, d\theta$

$$= \int \frac{4\sec^2\theta \, d\theta}{(16\tan^2\theta + 16)^2} = \int \frac{4}{16^2} \cdot \frac{\sec^2\theta}{\sec^4\theta} \, d\theta$$

$$= \frac{1}{64} \int \frac{1}{\sec^2\theta} \, d\theta = \frac{1}{64} \int \cos^2\theta \, d\theta$$

★ Half angle identity

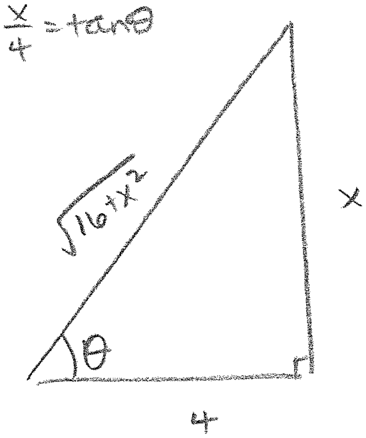
$$= \frac{1}{64} \int \frac{1}{2}(1 + \cos 2\theta) \, d\theta$$

$$= \frac{1}{128}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$

★ double angle identity

$$= \frac{1}{128}\left[\theta + \frac{2\sin\theta\cos\theta}{2}\right] + C$$

$\frac{x}{4} = \tan\theta$



$$= \frac{1}{128}\left[\arctan\left(\frac{x}{4}\right) + \frac{x}{\sqrt{16+x^2}} \cdot \frac{4}{\sqrt{16+x^2}}\right] + C$$