Math 182 Quiz **Name:** Key

2 Mar 2018 Point values in $\boxed{\text{boxes}}$.

1. $\boxed{3}$ Evaluate the following improper integral or show that it diverges:

$$\int_{-\infty}^{0} \frac{x}{x^4+1}\,dx.$$

$u = x^2$
$du = 2x\,dx$

$$= \lim_{R\to-\infty} \int_R^0 \frac{x}{(x^2)^2+1}\,dx = \lim_{R\to-\infty} \int_{R^2}^0 \frac{1}{2}\,\frac{1}{u^2+1}\,du$$

$$= \lim_{R\to-\infty} \frac{1}{2}\arctan u \Big|_{R^2}^{0} = \lim_{R\to-\infty} 0 - \frac{1}{2}\arctan(R^2)$$

$$= -\frac{1}{2}\cdot\frac{\pi}{2} = \boxed{\frac{-\pi}{4}}$$

2. $\boxed{3}$ Use the Comparison Theorem to determine if the following integral converges or diverges:

$$\int_0^{\pi/2} \frac{\sin^2(2x)}{x^{1/3}}\,dx.$$

$$0 \le \sin^2(2x) \le 1$$

so $0 \le \dfrac{\sin^2(2x)}{x^{1/3}} \le \dfrac{1}{x^{1/3}}$

Since $\int_0^{\pi/2} \frac{1}{x^{1/3}}\,dx$ converges ($p = 1/3 < 1$), then

$\int_0^{\pi/2} \frac{\sin^2(2x)}{x^{1/3}}\,dx$ must converge.

Continued on the reverse.

3. $\boxed{4}$ Find the surface area for the solid generated by rotating $y = \sqrt{1+e^x}$ for $x \in [0,1]$ about the x-axis.

$$SA = \int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

Aside

$$y = \sqrt{1+e^x}$$

① $y' = \frac{1}{2\sqrt{1+e^x}} \cdot e^x$

② $(y')^2 = \frac{e^{2x}}{4(1+e^x)}$

③ $(y')^2+1 = \frac{e^{2x}+4e^x+4}{4(1+e^x)}$

$$= \frac{(e^x+2)^2}{4(1+e^x)}$$

$$= \int_0^1 2\pi\sqrt{1+e^x}\sqrt{\frac{(e^x+2)^2}{4(1+e^x)}}\,dx$$

$$= \int_0^1 2\pi\sqrt{1+e^x}\,\frac{(e^x+2)}{2\sqrt{1+e^x}}\,dx$$

$$= \pi\int_0^1 e^x+2\,dx$$

$$= \pi\left[(e^x+2x)\Big|_0^1\right]$$

$$= \pi\left[e+2-1\right]$$

$$\boxed{= \pi\left[e+1\right]}$$