Math 182
9 Mar 2018

Quiz 6
Sections: 10.1/10.2

Name: ______________
Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ Let S_N be the N^{th} partial sum of $\sum_{n=1}^{\infty} a_n$. Please indicate **T**rue or **F** false.

   (a) **T** / **F** : If $a_n \to 0$ as $n \to \infty$, then a_n converges.
   (b) **T** / **F**: If $a_n \to 0$ as $n \to \infty$, then $\sum a_n$ converges.
   (c) **T** / **F** : If $S_N \to 0$ as $N \to \infty$, then $\sum a_n$ converges.
   (d) **T** / **F**: If $a_n \to 2$ as $n \to \infty$, then $\sum a_n$ converges.

2. $\boxed{3}$ Consider the series $\sum_{n=1}^{\infty} \arctan\left(\frac{1}{n^2+n+1}\right)$.

   (a) Find and simplify the partial sums S_2, S_3 and S_N.

   HINT: Trigonometry is weird, but: $\arctan\left(\frac{1}{n^2+n+1}\right) = \arctan(n+1) - \arctan n$

$S_2 = \arctan(2) - \arctan(1) + \arctan(3) - \arctan(2)$

$= \arctan 3 - \arctan 1$

$S_3 = \arctan 4 - \arctan 1$

$S_N = \arctan(N+1) - \arctan 1$

   (b) Find the sum of the series, or show that it diverges.

$S_N \xrightarrow[N\to\infty]{} \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

3. $\boxed{2}$ Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{1+(-2)^n}{3^n}.$$

$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n + \sum_{n=0}^{\infty} \left(\frac{-2}{3}\right)^n = \frac{1}{1-1/3} + \frac{1}{1-(-2/3)} = \frac{3}{2} + \frac{3}{5} = \frac{21}{10}$

CONTINUED ON THE REVERSE.

4. $\boxed{3}$ Find the sum of the following series, or show that the series diverges.

(a)

$$\frac{3}{8} - \frac{9}{32} + \frac{27}{128} - \frac{81}{512} + \cdots$$

$$= \frac{3/8}{1-(-3/4)} = \frac{\frac{3}{8}}{7/4} = \frac{3}{8} \cdot \frac{4}{7} = \frac{3}{14}$$

(b)

$$1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \frac{81}{16} + \frac{243}{32} + \cdots$$

Diverges since $r = \frac{3}{2}$

(c)

$$\frac{1}{9} + \frac{8}{9^2} + \frac{8^2}{9^3} + \frac{8^3}{9^4} + \frac{8^4}{9^4} + \frac{8^5}{9^6} + \cdots$$

$$= \frac{\frac{1}{9}}{1 - 8/9} = 1$$

5. $\boxed{1}$ Let SC_0 be a unit square. Subdivide SC_0 into nine congruent subsquares and remove the middle one, resulting in SC_1. Subdivide the remaining eight subsquares in SC_1 into nine congruent subsquares and remove the middle of each, generating SC_2. The limit of this process is the Sierpiński carpet, SC. The area removed is given by the series

$$\frac{1}{9} + \frac{8}{9^2} + \frac{8^2}{9^3} + \frac{8^3}{9^4} + \frac{8^4}{9^4} + \frac{8^5}{9^6} + \cdots.$$

Find the area of SC (see above).

$$SC = 1 - \sum_{n=0}^{\infty} \frac{1}{9}\left(\frac{8}{9}\right)^n = 1 - 1 = 0$$