

1. 2 Let S_N be the N^{th} partial sum of $\sum_{n=1}^{\infty} a_n$. Please indicate **T** or **F** false.

- (a) T / **F** : If $a_n \rightarrow 0$ as $n \rightarrow \infty$, then a_n converges.
 (b) **T** / F : If $a_n \rightarrow 0$ as $n \rightarrow \infty$, then $\sum a_n$ converges.
 (c) T / **F** : If $S_N \rightarrow 0$ as $N \rightarrow \infty$, then $\sum a_n$ converges.
 (d) **T** / F : If $a_n \rightarrow 2$ as $n \rightarrow \infty$, then $\sum a_n$ converges.

2. 3 Consider the series $\sum_{n=1}^{\infty} \arctan\left(\frac{1}{n^2 + n + 1}\right)$.

(a) Find and simplify the partial sums S_2 , S_3 and S_N .

HINT: Trigonometry is weird, but: $\arctan\left(\frac{1}{n^2 + n + 1}\right) = \arctan(n+1) - \arctan n$

$$S_2 = \arctan(2) - \arctan(1) + \arctan(3) - \arctan(2)$$

$$= \arctan(3) - \arctan(1)$$

$$S_3 = \arctan(4) - \arctan(1)$$

$$S_N = \arctan(N+1) - \arctan(1)$$

(b) Find the sum of the series, or show that it diverges.

$$S_N \xrightarrow{N \rightarrow \infty} \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

3. 2 Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{1 + (-2)^n}{3^n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n + \sum_{n=0}^{\infty} \left(\frac{-2}{3}\right)^n = \frac{1}{1 - 1/3} + \frac{1}{1 - (-2/3)} = \frac{3}{2} + \frac{3}{5} = \frac{21}{10}$$

4. **3** Find the sum of the following series, or show that the series diverges.

(a)

$$\begin{aligned} & \frac{3}{8} - \frac{9}{32} + \frac{27}{128} - \frac{81}{512} + \dots \\ &= \frac{3/8}{1 - (-3/4)} = \frac{3/8}{7/4} = \frac{3}{8} \cdot \frac{4}{7} = \frac{3}{14} \end{aligned}$$

(b)

$$1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \frac{81}{16} + \frac{243}{32} + \dots$$

Diverges since $r = \frac{3}{2}$

(c)

$$\frac{1}{9} + \frac{8}{9^2} + \frac{8^2}{9^3} + \frac{8^3}{9^4} + \frac{8^4}{9^5} + \frac{8^5}{9^6} + \dots$$

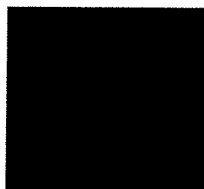
$$= \frac{\frac{1}{9}}{1 - 8/9} = 1$$

5. **1** Let SC_0 be a unit square. Subdivide SC_0 into nine congruent subsquares and remove the middle one, resulting in SC_1 . Subdivide the remaining eight subsquares in SC_1 into nine congruent subsquares and remove the middle of each, generating SC_2 . The limit of this process is the Sierpiński carpet, SC . The area removed is given by the series

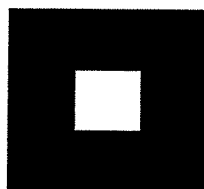
$$\frac{1}{9} + \frac{8}{9^2} + \frac{8^2}{9^3} + \frac{8^3}{9^4} + \frac{8^4}{9^5} + \frac{8^5}{9^6} + \dots$$

Find the area of SC (see above).

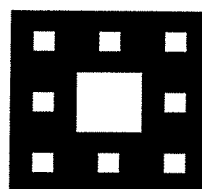
$$SC = 1 - \sum_{n=0}^{\infty} \frac{1}{9} \left(\frac{8}{9}\right)^n = 1 - 1 = 0$$



SC_0

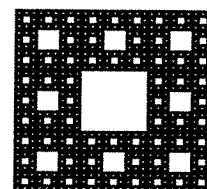


SC_1



SC_2

...



SC