Math 182 Quiz 7
22 Mar 2018

Name: key

Point values in $\boxed{\text{boxes}}$.

1. $\boxed{1}$ For the following series, specify what series you would compare each to (using either the direct or limit comparison test), and then based on your comparison, decide if the series converges or diverges. No formal justification is required.

   (a) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{2+\sin^2 n}\right)^n$ compare to $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$ so it CONVERGES/DIVERGES.

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{3}{n^2+\sqrt{n}}$ compare to $\displaystyle\sum_{n=1}^{\infty}\frac{3}{n^2}$ so it CONVERGES/DIVERGES.

2. $\boxed{1}$ For each statement, determine if the use of the Comparison Test is **V**alid or **I**nvalid.

   (a) **V** / **I**: Since $0<\dfrac{1}{n+3}<\dfrac{1}{n}$ and $\sum\dfrac{1}{n}$ diverges, by comparison $\sum\dfrac{1}{n+3}$ also diverges.

   (b) **V** / **I**: Since $0<\dfrac{1}{n^2}<\dfrac{1}{n^2-1}$ and $\sum\dfrac{1}{n^2}$ converges, by comparison $\sum\dfrac{1}{n^2-1}$ also converges.

3. $\boxed{2}$ State all of the hypotheses that must be met in order to use the Integral Test.

   $f(x)$, where $f(n)=a_n$, must be positive, continuous, and decreasing on the specified interval.

4. $\boxed{1}$ For each of the following series, determine if the Test for Divergence can be used to show the series **C**onverges, **D**iverges, or is **I**nconclusive.

   (a) **C** / **D** / **I**: $\sum\dfrac{1}{n}$

   (b) **C** / **D** / **I**: $\sum\dfrac{1}{n^2}$

   (c) **C** / **D** / **I** : $\sum\dfrac{1}{1+\left(\frac{2}{3}\right)^n}$

   (d) **C** / **D** / **I** : $\sum\sin n$

CONTINUED ON THE REVERSE.

6. $\boxed{5}$ Determine whether each series is convergent or divergent. Indicate which test you are using, verify hypotheses, and state clear conclusions.

(a) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{1+n^2}$

Direct Comparison test

$$0 \le \frac{\cos^2 n}{1+n^2} \le \frac{1}{n^2}$$

Because $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges $(p = 2 > 1)$, then by the Direct Comparison Test,

$\sum_{n=1}^{\infty} \frac{\cos^2 n}{1+n^2}$ must converge.

(b) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^{3/2}}$

Direct Comparison test

$$0 \le \frac{\sqrt{n}}{1+n^{3/2}} \le \frac{\sqrt{n}}{n^{3/2}} = \frac{1}{n}$$

no conclusion can be made

Limit Comp. Test

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n}}{1+n^{3/2}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^{3/2}}{1+n^{3/2}} = 1 > 0$$

Because $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, by the LCT, $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^{3/2}}$ must also diverge (the series behave the same).