Math 182 Quiz 8 **Name:** key

29 Mar 2018 Point values in $\boxed{\text{boxes}}$.

1. $\boxed{1}$ Please indicate **T**rue or **F** false.

   (a) **T** / **F** : If $\sum |a_n|$ converges then $\sum a_n$ converges.

   (b) **T** / **F**: If $\sum |a_n|$ diverges then $\sum a_n$ converges conditionally. only if Σa_n converges

2. $\boxed{1}$ Find the error bound specified by the Alternating Series Test for

$$\left|\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}-\sum_{n=0}^{4}\frac{(-1)^n}{n!}\right| < \underline{\frac{1}{5!}}$$

$|S - S_4| < b_5 \qquad b_n = \frac{1}{n!}$

3. $\boxed{8}$ Determine if the following converge conditionally, converge absolutely, or diverge. Provide appropriate justification.

   (a) $\displaystyle\sum_{n=3}^{\infty}\frac{4\cos(5n)-3}{n^2+3n}$

Consider $\displaystyle\sum_{n=3}^{\infty}\left|\frac{4\cos(5n)-3}{n^2+3n}\right|$

$$0 \le \frac{|4\cos(5n)-3|}{n^2+3n} \le \frac{7}{n^2}$$

Since $\displaystyle\sum_{n=3}^{\infty}\frac{7}{n^2}$ converges $(p=2>1)$, then $\displaystyle\sum_{n=3}^{\infty}\frac{4\cos(5n)-3}{n^2+3n}$ must converge by Direct Comparison.

   (b) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{\sqrt{n^2+3n}}$

① Consider $\displaystyle\sum_{n=2}^{\infty}\left|\frac{(-1)^n}{\sqrt{n^2+3n}}\right| = \sum_{n=2}^{\infty}\frac{1}{\sqrt{n^2+3n}}$ $\qquad a_n = \frac{1}{\sqrt{n^2+3n}} > 0,\ b_n = \frac{1}{n} > 0$

LCT: $\displaystyle\lim_{n\to\infty}\frac{\frac{1}{\sqrt{n^2+3n}}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{n}{\sqrt{n^2+3n}} = 1 > 0$

Since $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n}$ diverges (Harmonic), by LCT, $\displaystyle\sum_{n=2}^{\infty}\frac{1}{\sqrt{n^2+3n}}$ diverges as well.

② Since $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{\sqrt{n^2+3n}}$ converges by AST
$\left(\begin{array}{l} ①\ b_n = \frac{1}{\sqrt{n^2+3n}} > 0 \\ ②\ \lim_{n\to\infty}\frac{1}{\sqrt{n^2+3n}} = 0 \\ ③\ b_{n+1} = \frac{1}{\sqrt{(n+1)^2+3(n+1)}} < \frac{1}{\sqrt{n^2+3n}} = b_n \end{array}\right)$

then $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{\sqrt{n^2+3n}}$ is conditionally convergent.