Math 182 Quiz 9
6 Apr 2018

Name: ____________________

Point values in $\boxed{\text{boxes}}$.

1. $\boxed{1}$ Please indicate **T**rue or **F** false. Center = -2 Radius ≥ 4

   (a) T / F : Suppose $\sum a_n(x+2)^n$ converges when $x = 2$. Then it must converge when $x = -4$.

   (b) T / F: Suppose $\sum a_n(x+2)^n$ converges when $x = 0$. Then it must converge when $x = -4$.

2. $\boxed{1}$ For a power series $\sum a_n(x-c)^n$, suppose $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0$.
   State the radius and interval of convergence.

   Radius = ∞ , Interval = $\mathbb{R}$

3. $\boxed{5}$ For the power series below, find and clearly state the radius and interval of convergence. Provide appropriate justification.

$$\sum_{n=3}^{\infty} \frac{(-1)^n(x-1)^n}{2^n n^2}$$

$$\sqrt[n]{\left|\frac{(-1)^n(x-1)^n}{2^n \cdot n^2}\right|} = \frac{|x-1|}{2}\cdot\sqrt[n]{\frac{1}{n^2}} \xrightarrow[n\to\infty]{} \frac{|x-1|}{2} < 1$$

so $\sum_{n=3}^{\infty}\frac{(-1)^n(x-1)^n}{2^n n^2}$ converges absolutely when $|x-1| < 2$,

i.e. the radius of convergence is $\underline{\underline{2}}$.

To find the interval we check the endpoints

$x = 3$: $\sum_{n=3}^{\infty}\frac{(-1)^n 2^n}{2^n n^2} = \sum_{n=3}^{\infty}\frac{(-1)^n}{n^2}$ which converges absolutely since $\sum_{n=3}^{\infty}\frac{1}{n^2}$ converges,

$x = -1$: $\sum_{n=3}^{\infty}\frac{(-1)^n(-2)^n}{2^n n^2} = \sum_{n=3}^{\infty}\frac{1}{n^2}$ which is a convergent p-series,

so the interval of convergence is $[-1, 3]$.

Continued on the reverse.

4. $\boxed{3}$ Use the geometric series $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $|x| < 1$ to find a series representation for $f(x) = -\ln(1-x)$ centered at $c = 0$ and its interval of convergence.

HINT: $f'(x) = \frac{1}{1-x}$.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for} \quad |x| < 1$$

so

$$-\ln(1-x) = \int \frac{1}{1-x}\,dx + A \quad \text{for} \quad |x|<1$$

$$= \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} + A \quad \text{for} \quad |x|<1$$

Let $x = 0$.

$$-\ln(1) = A \quad \text{so} \quad A = 0,$$

so

$$-\ln(1-x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} \quad \text{for} \quad |x| < 1.$$

Does the series converge for $x = \pm 1$?

$x = 1$: $\sum_{n=0}^{\infty} \frac{1}{n+1}$ diverges since $\sum_{n=0}^{\infty} \frac{1}{n+1} = \sum_{K=1}^{\infty} \frac{1}{K}$ is the Harmonic.

$x = -1$: $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ converges by the Alternating Series Test

since $\frac{1}{n+1} > 0$, $\frac{1}{n+1} > \frac{1}{n+2}$, and $\frac{1}{n+1} \xrightarrow[n\to\infty]{} 0$.