Review of Series Tests and Facts

TEST	SERIES	CONDITIONS	RESULT
	Geometric	For $c \neq 0$, $\sum_{n=M}^{\infty} cr^n$ $= cr^M + cr^{M+1} + cr^{M+2} + \ldots$	Conv. for $\|r\| < 1$ to $\frac{cr^M}{1-r}$. Diverges for $\|r\| \geq 1$.
	P-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges for $p > 1$. Diverges for $p \leq 1$.
	Harmonic	$\sum_{n=1}^{\infty} \frac{1}{n}$ $(p = 1)$	Diverges
Divergence Test	Any Series	$\lim_{n\to\infty} a_n \neq 0$	$\sum a_n$ diverges
Integral Test	Positive	$f(x)$ is positive, cont. & decreasing where $f(n) = a_n$	If $\int_*^{\infty} f(x)\,dx$ converges then $\sum_{n=*}^{\infty} a_n$ converges. If $\int_*^{\infty} f(x)\,dx$ diverges then $\sum_{n=*}^{\infty} a_n$ diverges.
Comparison Test	Positive	$0 \leq a_n \leq b_n$ & $\sum b_n$ conv.	$\sum a_n$ converges
		$0 \leq b_n \leq a_n$ & $\sum b_n$ div.	$\sum a_n$ diverges
Limit Comparison Test	Positive	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$	$\sum a_n$ and $\sum b_n$ have the same behavior
		$\lim_{n\to\infty} \frac{a_n}{b_n} = L = \infty$	If $\sum b_n$ div., $\sum a_n$ div.
		$\lim_{n\to\infty} \frac{a_n}{b_n} = L = 0$	If $\sum b_n$ conv., $\sum a_n$ conv.
Alternating Series Test	$\sum (-1)^n b_n$	$b_n > 0$, $\lim_{n\to\infty} b_n = 0$, and $b_{n+1} < b_n$	$\sum (-1)^n b_n$ converges.
Ratio Test	Any Series	$\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| = L$	Absolutely Conv: $L < 1$ Divergent: $L > 1$, or ∞ Inconclusive: $L = 1$
Root Test	Any Series	$\lim_{n\to\infty} \sqrt[n]{\|a_n\|} = L$	Absolutely Conv: $L < 1$ Divergent: $L > 1$, or ∞ Inconclusive: $L = 1$