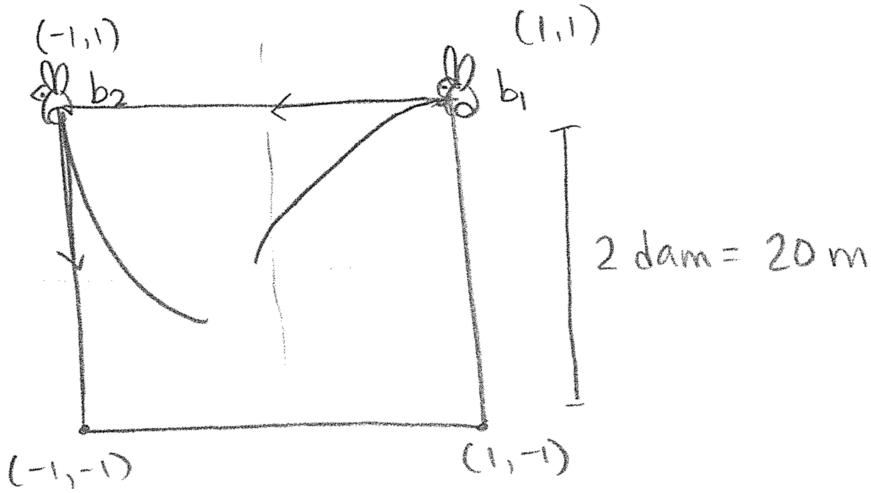
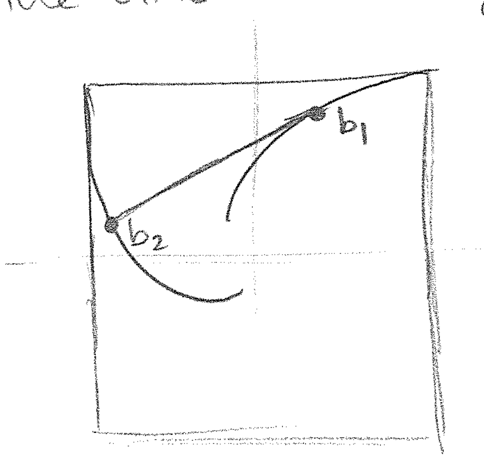
Polar Problems

1.



a) Find polar equations for each of the bunnies.

Consider the line connecting b_1 to b_2.



<u>Cartesian</u>:

if b_1 is at (x,y),
b_2 will be at $(-y,x)$

(consider a 90° rotation)

OR

<u>polar</u>:

If b_1 is at $(r\cos\theta, r\sin\theta)$
then b_2 is at $(\underbrace{r\cos(\theta+\pi/2)}_{\underline{-r\sin\theta}\;-y}, \underbrace{r\sin(\theta+\pi/2)}_{\underline{r\cos\theta}\;x})$

Using polar, slope of $\overline{b_1b_2}$ will be $\dfrac{r\sin\theta - r\cos\theta}{r\cos\theta - -r\sin\theta}$

1a)

$$\frac{r\sin\theta - r\cos\theta}{r\cos\theta + r\sin\theta} = \frac{r\cos\theta - r\sin\theta}{-r\sin\theta - r\cos\theta}$$

- if $r' = -r$, then the above slope matches the definition for polar slope:

$$\frac{r\cos\theta - r\sin\theta}{-r\sin\theta - r\cos\theta} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta} \checkmark$$

Since $r = Ce^{-\theta}$ solves $r' = -r$, we are left to find the equations for all four bunnies' paths.

$\underline{b_1}$:

- point $(1,1)$ in cartesian is $(\sqrt{2}, \pi/4)$ in polar

$$\left[\begin{array}{l} x^2+y^2 = 1^2+1^2 = r^2 \\ \tan^{-1}(y/x) = \tan^{-1}(1) = \theta \end{array}\right]$$

$$r_1 = C_1 e^{-\theta}$$

$$\sqrt{2} = C_1 e^{-\pi/4}$$

$$C_1 = \frac{\sqrt{2}}{e^{-\pi/4}} = \sqrt{2}\, e^{\pi/4}$$

$$r_1 = \sqrt{2}\, e^{\pi/4} e^{-\theta} = \sqrt{2}\, e^{-\theta + \pi/4}$$

Similarly,

$$r_2 = \sqrt{2}\, e^{-\theta + 3\pi/4}$$

$$r_3 = \sqrt{2}\, e^{-\theta + 5\pi/4}$$

$$r_4 = \sqrt{2}\, e^{-\theta + 7\pi/4}$$

b) Find the arclength of each path. Consider the path of bunny 1:

$$S = \int_a^b \sqrt{r^2 + (r')^2}\, d\theta = \int_{\pi/4}^{\infty} \sqrt{r^2 + r^2}\, d\theta \quad \text{Since } r = r'$$

$$= \int_{\pi/4}^{\infty} \sqrt{2}\, r\, d\theta = \int_{\pi/4}^{\infty} \sqrt{2} \cdot \sqrt{2}\, e^{-\theta + \pi/4}\, d\theta = -2e^{-\theta+\pi/4} \Big|_{\pi/4}^{\infty}$$

$$= \lim_{R \to \infty} \left(-2e^{\pi/4} e^{-R} + 2e^{\pi/4} e^{-\pi/4} \right) = 2 \text{ dam}$$

c) $20\text{ m} = 4\,\frac{\text{m}}{\text{s}} \cdot x$, $x = 5\text{ s}$

d) After an infinite number of revolutions in 5s, the bunnies are very dizzy!