Math 182 Exam 2 **Name:** Key

20 Feb 2018 Point values in $\boxed{\text{boxes}}$.

1. Consider the region bounded by the graph of $y = \sin 2x$ and the x-axis for $x \in [0, \frac{\pi}{2}]$.

   (a) $\boxed{10}$ Rotating the region about the x-axis generates a solid with volume given by the integral

   $$\pi \int_0^{\pi/2} \sin^2 2x \, dx,$$

   find the volume.

   $$= \pi \int_0^{\pi/2} \frac{1}{2}(1 - \cos 4x)\, dx$$

   $$= \frac{\pi}{2}\left[\left(x - \frac{\sin 4x}{4}\right)\Big|_0^{\pi/2}\right]$$

   $$= \frac{\pi}{2}\left[\pi/2 - \frac{\sin 2\pi}{4} - (0-0)\right]$$

   $$= \frac{\pi^2}{4}$$

   (b) $\boxed{10}$ Rotating the region about the line $x = \pi$ generates a solid with volume given by the integral

   $$2\pi \int_0^{\pi/2} (\pi - x) \sin 2x \, dx,$$

   find the volume.

   IBP

   $u = \pi - x \quad dv = \sin 2x\, dx$

   $du = -dx \quad v = \frac{-\cos 2x}{2}$

   $$= 2\pi\left[\frac{-(\pi - x)\cos(2x)}{2}\Big|_0^{\pi/2} - \int_0^{\pi/2} \frac{1}{2}\cos(2x)\, dx\right]$$

   $$= 2\pi\left[-\frac{(\pi - \pi/2)(-1)}{2} + \frac{(\pi)(1)}{2} - \underbrace{\left(\frac{1}{4}\sin(2x)\Big|_0^{\pi/2}\right)}_{0}\right]$$

   $$= 2\pi\left[\pi/4 + \pi/2\right] = \frac{3\pi^2}{2}$$

2. Evaluate.

(a) $\boxed{10}$ $\int \sec^8 t \tan^3 t\,dt = \int \sec^6 t \tan^3 t \sec^2 t\,dt$ OR $= \underbrace{\int \sec^7 t \tan^2 t \sec t \tan t\,dt}_{\text{easier}}$

$u = \sec t$
$du = \sec t \tan t\,dt$

$= \int \sec^7 t\,(\sec^2 t - 1) \sec t \tan t\,dt$

$= \int u^7 (u^2 - 1)\,du = \int (u^9 - u^7)\,du$

$= \frac{u^{10}}{10} - \frac{u^8}{8} + C = \frac{\sec^{10} t}{10} - \frac{\sec^8 t}{8} + C$

(b) $\boxed{20}$ $\int \frac{5x^3 - 3x^2 + 15x + 5}{\underbrace{x^4 + 5x^2}_{x^2(x^2+5)}}\,dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2+5}\,dx$

Finding coefficients

$5x^3 - 3x^2 + 15x + 5 = Ax(x^2+5) + B(x^2+5) + (Cx+D)x^2$

let $x = 0$: $5 = B(0+5)$
$B = 1$

$5x^3 - 3x^2 + 15x + 5 = Ax^3 + 5Ax + x^2 + 5 + Cx^3 + Dx^2$

$$\left.\begin{array}{l} A + C = 5 \\ D + 1 = -3 \\ 5A = 15 \\ 5 = 5 \checkmark \end{array}\right\}$$

$A = 3,\ C = 2$
$D = -4$

$\int \frac{3}{x} + \frac{1}{x^2} + \frac{2x-4}{x^2+5}\,dx = 3\ln|x| - \frac{1}{x} + \int \frac{2x}{x^2+5}\,dx - \int \frac{4}{x^2+5}\,dx$

$u = x^2 + 5$
$du = 2x\,dx$

* see last page of test

$= 3\ln|x| - \frac{1}{x} + \ln|x^2+5| - \frac{4}{\sqrt{5}} \arctan(x/\sqrt{5}) + C$

3. $\boxed{25}$ Evaluate

$$\int \frac{x^2\,dx}{\sqrt{x^2+9}}.$$

let $x = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta$

$$= \int \frac{9\tan^2\theta \cdot 3\sec^2\theta\,d\theta}{\sqrt{9\tan^2\theta+9}}$$

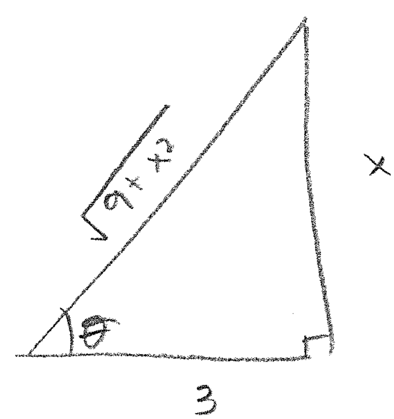
$$= \frac{27}{3}\int \frac{\tan^2\theta\,\sec^2\theta}{\sqrt{\sec^2\theta}}\,d\theta = 9\int \tan^2\theta\,\sec\theta\,d\theta$$

$$= 9\int(\sec^2\theta-1)\sec\theta\,d\theta = 9\int\underbrace{(\sec^3\theta-\sec\theta)}_{\text{from last page of test}}\,d\theta$$

$$= 9\left[\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta| - \ln|\sec\theta+\tan\theta|\right]+C$$

$$= \frac{9}{2}\left[\sec\theta\tan\theta - \ln|\sec\theta+\tan\theta|\right] + C$$

$$= \frac{9}{2}\left[\frac{x\sqrt{9+x^2}}{9} - \ln\left|\frac{\sqrt{9+x^2}}{3}+\frac{x}{3}\right|\right]+C$$



4. $\boxed{25}$

$$\int \frac{\arcsin x}{x^2}\,dx$$

inverse trig → try IBP

$u = \arcsin x \qquad dv = \frac{1}{x^2}\,dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \qquad v = \frac{-1}{x}$

$$\int \frac{\arcsin x}{x^2}\,dx = -\frac{\arcsin x}{x} + \underbrace{\int \frac{1}{x\sqrt{1-x^2}}\,dx}_{\text{trig sub}}$$

let $x = \sin\theta$

$dx = \cos\theta\, d\theta$

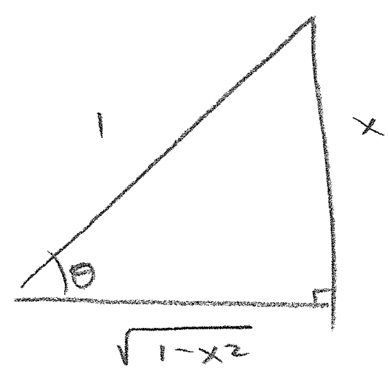
$$= -\frac{\arcsin x}{x} + \int \frac{\cos\theta\, d\theta}{\sin\theta\sqrt{1-\sin^2\theta}}$$

$$= -\frac{\arcsin x}{x} + \int \frac{\cos\theta}{\sin\theta\sqrt{\cos^2\theta}}\,d\theta$$

$$= -\frac{\arcsin x}{x} + \int \frac{1}{\sin\theta}\,d\theta = -\frac{\arcsin x}{x} + \underbrace{\int \csc\theta\, d\theta}_{\text{see last page of test}}$$

$$= -\frac{\arcsin x}{x} + \ln\left|\csc\theta - \cot\theta\right| + C$$

$$= -\frac{\arcsin x}{x} + \ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + C$$



Problem	1	2	3	4	Total
Value	20	30	25	25	100
Points					

Some trigonometric identities.

$$\begin{aligned}\cos^2 x &= \frac{1}{2}(1+\cos 2x)\\ \sin^2 x &= \frac{1}{2}(1-\cos 2x)\\ \sin 2x &= 2\ \sin x\ \cos x\end{aligned}$$

Some integrals.

$$\begin{aligned}\int \frac{du}{a^2+u^2} &= \frac{1}{a}\arctan\left(\frac{u}{a}\right)+c\\ \int \frac{du}{\sqrt{a^2-u^2}} &= \arcsin\left(\frac{u}{a}\right)+c\\ \int \sec u\ du &= \ln|\sec u+\tan u|+c\\ \int \sec^3 u\ du &= \frac{1}{2}\sec u\tan u+\frac{1}{2}\ln|\sec u+\tan u|+c\\ \int \csc u\ du &= \ln|\csc u-\cot u|+c\end{aligned}$$