Math 182 Exam 4 **Name:** Key

16 April 2018 Point values in $\boxed{\text{boxes}}$.

1. $\boxed{20}$ Find and clearly state the interval of convergence for

$$\sum_{n=1}^{\infty} \frac{(-2)^n(x+1)^n}{4n-3}.$$

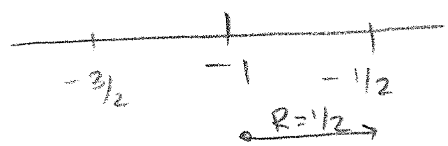
Formal justification is required.

Ratio Test:

$$\lim_{n\to\infty} \left| \frac{(-2)^{n+1}(x+1)^{n+1}}{4(n+1)-3} \frac{4n-3}{(-2)^n(x+1)^n} \right| = \lim_{n\to\infty} 2|x+1| \cdot \frac{4n-3}{4n+1}$$

$$= 2|x+1| < 1 \quad \Rightarrow \quad |x+1| < 1/2 \text{ and } R = 1/2.$$

Test end points:



let $x = -1/2$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n(1/2)^n}{4n-3} = \sum_{n=1}^{\infty} \frac{(-1)^n}{4n-3}$

AST: $b_n = \frac{1}{4n-3} > 0$, $\lim_{n\to\infty} \frac{1}{4n-3} = 0$, $b_{n+1} = \frac{1}{4n+1} < \frac{1}{4n-3} = b_n$

this series converges.

let $x = -3/2$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n(-1/2)^n}{4n-3} = \sum_{n=1}^{\infty} \frac{1}{4n-3}$

LCT: $b_n = \frac{1}{n} > 0$, $a_n = \frac{1}{4n-3} > 0$

$$\lim_{n\to\infty} \frac{\frac{1}{4n-3}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{4n-3} = \frac{1}{4} > 0.$$

since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, $\sum_{n=1}^{\infty} \frac{1}{4n-3}$ diverges by LCT.

Interval is $(-3/2, -1/2]$.

2. For $c > 0$ consider the function

$$f(x) = \frac{3c}{(2+cx)^2}.$$

HINT: $\frac{d}{dx}\left(\frac{3}{2+cx}\right) = \frac{-3c}{(2+cx)^2}$

(a) $\boxed{12}$ Find a power series representation for $f(x)$.

$$\frac{3c}{(2+cx)^2} = \frac{d}{dx}\left(\frac{-3}{2+cx}\right) = \frac{d}{dx}\left(\frac{-3}{2}\cdot\frac{1}{1+\frac{cx}{2}}\right) = \frac{d}{dx}\left(\frac{-3}{2}\cdot\frac{1}{1-(-\frac{cx}{2})}\right)$$

$$= \frac{d}{dx}\left(\sum_{n=0}^{\infty}\frac{-3}{2}\left(\frac{-cx}{2}\right)^n\right)$$

$$= \sum_{n=1}^{\infty} +\frac{3nc}{4}\left(-\frac{cx}{2}\right)^{n-1}$$

(b) $\boxed{3}$ Find the radius of convergence (not the interval) for the series representation in (a).

$$\left|\frac{-cx}{2}\right| < 1 \quad \Rightarrow \quad |x| < \frac{2}{c} \quad \text{so} \quad R = \frac{2}{c}$$

3. Consider the function

$$g(x) = \int_0^x \sin(t^2)\, dt.$$

(a) $\boxed{12}$ Find a power series representation for $g(x)$.

$$g(x) = \int_0^x \sum_{n=0}^{\infty}\frac{(-1)^n(t^2)^{2n+1}}{(2n+1)!}\,dt = \int_0^x \sum_{n=0}^{\infty}\frac{(-1)^n t^{4n+2}}{(2n+1)!}\,dt$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n t^{4n+3}}{(4n+3)(2n+1)!}\Bigg|_0^x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}$$

(b) $\boxed{3}$ Evaluate $g^{(2018)}(0)$.

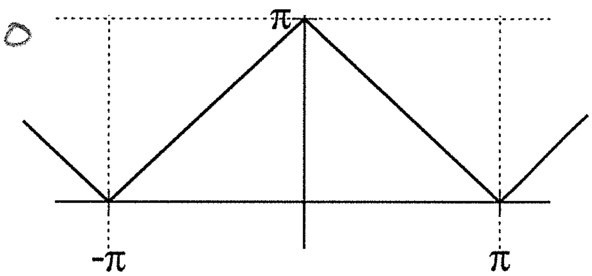
$$\underbrace{g^{(2018)}(0)}_{(2018^{\text{th}}\text{ derivative eval. at } 0)} = 2018!\, a_{2018} = 0$$ because no even coefficients

4. $\boxed{20}$ Find the coefficients (a_0, a_n, and b_n) for the Fourier series expansion of

$$f(x) = \begin{cases} x+\pi, & -\pi \le x \le 0 \\ \pi - x, & 0 \le x \le \pi \end{cases}$$

where $f(x) = f(x+2\pi)$ for all x, i.e. f is 2π periodic. See figure.

even function so $b_n = 0$



$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0}(x+\pi)\,dx + \int_0^\pi (\pi - x)\,dx\right]$$

$$= \frac{2}{2\pi}\int_0^\pi (\pi - x)\,dx = \frac{1}{\pi}\left[\left(\pi x - \frac{x^2}{2}\right)\Big|_0^\pi\right] = \frac{1}{\pi}\left[\frac{\pi^2}{2}\right] = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(x+\pi)\cos(nx)\,dx + \int_0^\pi (\pi - x)\cos(nx)\,dx\right] = \frac{2}{\pi}\underbrace{\int_0^\pi (\pi - x)\cos(nx)\,dx}$$

IBP:

$u = \pi - x \quad dv = \cos nx\, dx$

$du = -dx \quad v = \frac{\sin nx}{n}$

$$= \frac{2}{\pi}\left[\underbrace{(\pi - x)\frac{\sin nx}{n}\Big|_0^\pi}_{0} + \int_0^\pi \frac{\sin nx}{n}\,dx\right]$$

$$= \frac{2}{\pi}\left[-\frac{\cos nx}{n^2}\Big|_0^\pi\right] = \frac{2}{\pi}\left[\frac{-\cos(n\pi)}{n^2} - -\frac{1}{n^2}\right]$$

$$= \frac{2}{\pi n^2}\left[1 - \cos(n\pi)\right] = \begin{cases} \frac{4}{\pi n^2} & \text{if } n \text{ odd} \\ 0 & \text{if } n \text{ even} \end{cases}$$

5. Fill in the following blanks. Work is not needed nor expected.

(a) $\boxed{3}$

$$\left|\underset{\arctan 1}{\frac{\pi}{4}} - \overbrace{\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}\right)}^{S_4}\right| < \overbrace{\underline{\frac{1}{11}}}^{b_5}$$

(b) $\boxed{2}$

$$\underset{\cos^2(c)}{\left(\sum_{n=0}^{\infty} \frac{(-1)^n c^{2n}}{(2n)!}\right)^2} + \underset{\sin^2(c)}{\left(\sum_{n=0}^{\infty} \frac{(-1)^n c^{2n+1}}{(2n+1)!}\right)^2} = \underline{\quad 1 \quad}$$

(c) $\boxed{2}$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \underline{\quad e \quad}$$

6. $\boxed{8}$ Please circle **True** or **False**, as appropriate.

(a) **T** / **F:** If $\lim_{n\to\infty} |a_n| = 0$, then $\sum a_n$ converges absolutely.

(b) **T** / **F:** There exists a sequence $\{c_n\}$ so that $\sum c_n x^n$ converges for $x \in [0, \infty)$ and diverges for $x \in (-\infty, 0)$.

(c) **T** / **F:** $\sum_{w=2}^{\infty} \frac{(-1)^w}{\ln w^4}$ converges conditionally.

(d) **T** / **F:** If $\sum |a_n|$ diverges, then $\sum a_n$ diverges.

7. $\boxed{15}$ Find the first three nonzero terms for the Maclaurin series of

$$e^{2x}\sin x - 2x^2.$$

HINT: Do not compute derivatives. Rather, use the provided tables.

$$e^{2x} = 1 + (2x) + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$e^{2x}\sin x = x + 2x^2 + \frac{4x^3}{2!} - \frac{x^3}{3!} + \frac{8x^4}{3!} - \frac{2x^4}{3!} + \cdots$$

$$e^{2x}\sin x - 2x^2 = x + \left(2 - \frac{1}{6}\right)x^3 + \left(\frac{8}{6} - \frac{2}{6}\right)x^4 + \cdots$$

$$= x + \frac{11}{6}x^3 + x^4 + \cdots$$

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Value	20	30	20	30	100
Points					