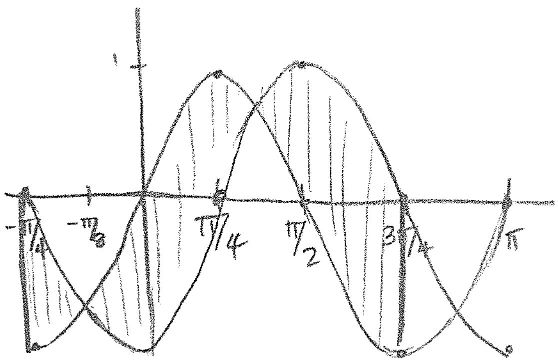
Math 182 Problems **Name:** Key

Sections: 6.1, 6.2

Due: 19 Jan 2018

Point values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ Carefully sketch a graph of $y = \sin 2x$ and $y = -\cos 2x$ for $x \in \left[\frac{-\pi}{4}, \frac{3\pi}{4}\right]$. Find the area between the graphs in the specified interval.



$$A = \int_{-\pi/4}^{-\pi/8} -\cos 2x - \sin 2x \, dx + \int_{\pi/8}^{3\pi/8} \sin 2x + \cos 2x \, dx$$

$$+ \int_{3\pi/8}^{3\pi/4} -\cos 2x - \sin 2x \, dx$$

$\sin 2x = -\cos 2x$

$\tan 2x = -1$

$2x = \arctan(-1) = -\pi/4$

$x = -\pi/8$

$\underline{\text{OR}}$ $$A = 2\int_{-\pi/8}^{3\pi/8} \sin 2x + \cos 2x \, dx$$

$$= 2\left[-\frac{\cos 2x}{2} + \frac{\sin 2x}{2}\Big|_{-\pi/8}^{3\pi/8}\right]$$

$$= -\left(-\sqrt{2}/2\right) + \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2} + -\frac{\sqrt{2}}{2}\right) = \boxed{2\sqrt{2}}$$

2. The birth rate of a population is $b(t) = 2200e^{0.022t}$ people per year, and the death rate is $d(t) = 1080e^{0.018t}$ people per year.

   (a) $\boxed{2}$ Find the area between the graphs of these functions for $0 \le t \le 10$.

$$\int_0^{10} 2200e^{0.022t} - 1080e^{0.018t} \, dt = \frac{2200e^{0.022t}}{0.022} - \frac{1080e^{0.018t}}{0.018}\Bigg|_0^{10}$$

$$= 100{,}000e^{0.22} - 60{,}000e^{0.18} - [100{,}000 - 60{,}000]$$

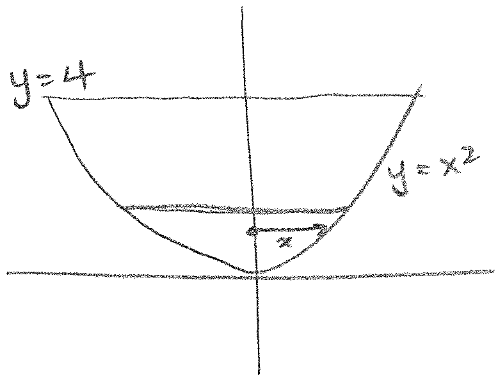
$$= 12{,}774.6$$

   (b) $\boxed{1}$ What does the area represent? (A sentence with appropriate punctuation is appropriate here.)

   The area represents the change in population over 10 years.

3. Find the volume of the described solids using appropriate integration techniques.

(a) $\boxed{2}$ The base is the region enclosed by the graph of $y = x^2$ and the graph of $y = 4$. Cross sections perpendicular to the y-axis are rectangles of height y^2.



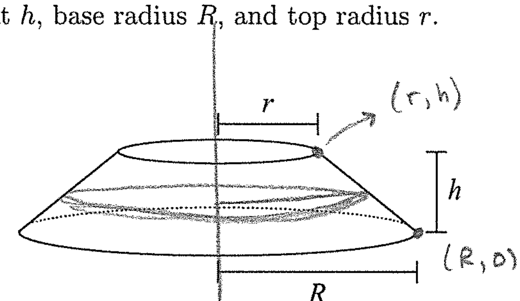
variable: y

$$A_{slice} = l \cdot w = (2x)y^2 = 2\sqrt{y}\,y^2$$

$$V = \int_0^4 2y^{5/2}\,dy = \frac{4y^{7/2}}{7}\Big|_0^4$$

$$= \frac{4}{7}\left(4^{7/2}\right) = \frac{512}{7}$$

(b) $\boxed{3}$ A frustum of a right circular cone with height h, base radius R, and top radius r.



relate x and y

$$m = \frac{h}{r-R}$$

$$y - 0 = \frac{h}{r-R}(x - R)$$

$$y = \frac{hx}{r-R} - \frac{hR}{r-R}$$

$$y + \frac{hR}{r-R} = \frac{h}{r-R}x$$

$$x = \frac{r-R}{h}\left[y + \frac{hR}{r-R}\right]$$

variable: y

$$A_{slice} = \pi x^2$$

$$= \pi\left[\frac{r-R}{h}\left(y + \frac{hR}{r-R}\right)\right]^2$$

$$V = \int_0^h \pi\left(\frac{r-R}{h}\right)^2\left(y + \frac{hR}{r-R}\right)^2 dy$$

$$= \pi\left(\frac{r-R}{h}\right)^2 \int_0^h y^2 + \frac{2hR}{r-R}y + \left(\frac{hR}{r-R}\right)^2 dy$$

$$= \pi\left(\frac{r-R}{h}\right)^2\left[\frac{y^3}{3} + \frac{hR}{r-R}y^2 + \left(\frac{hR}{r-R}\right)^2 y\Big|_0^h\right]$$

$$= \pi\left(\frac{r-R}{h}\right)^2\left[\frac{h^3}{3} + \frac{h^3R}{r-R} + \frac{h^3R^2}{(r-R)^2}\right]$$