M273Q, FALL 2016 Name:________________________________
Section: ___________
EXAM 1; September 27, 2016; 100 points

Instructions: Closed book. No calculator allowed. Double-sided exam. NO CELL PHONES. Show all work and use correct notation to receive full credit! Write legibly.

Problem	1	2	3	4	5	6	7	8	9	10	Total
Points Possible	4	4	4	9	14	10	10	15	15	15	100
Points Earned											

$$\kappa(s) = \left|\left|\frac{d\mathbf{T}}{ds}\right|\right| \qquad \kappa(x) = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}$$

$$\kappa(t) = \frac{||\mathbf{T}'(t)||}{||\mathbf{r}'(t)||} \qquad a_{\mathbf{N}} = \kappa(t)[v(t)]^2.$$

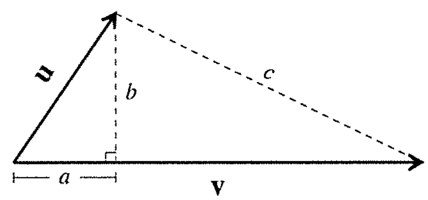
$$\kappa(t) = \frac{||\mathbf{r}'(t)\times\mathbf{r}''(t)||}{||\mathbf{r}'(t)||^3}$$

1. (4 points total) True or False? Circle ONE answer for each.
   (a) True or False: For any vectors $\mathbf{u}, \mathbf{v}$ in $\mathbb{R}^3$, $\mathbf{u}\times\mathbf{v} = \mathbf{v}\times\mathbf{u}$. $\vec{u}\times\vec{v} = -(\vec{v}\times\vec{u})$
   (b) True or False: If $||\mathbf{r}(t)|| = 1$ for all t, then $||\mathbf{r}'(t)||$ is a constant. An ant on a pool ball does not have to have a constant speed
   (c) True or False: The binormal vector is $\mathbf{B}(t) = \mathbf{N}(t)\times\mathbf{T}(t)$. $\vec{B} = \vec{T}\times\vec{N}$
   (d) True or False: A linear equation $ax + by + cz + d = 0$ represents a line in space. This is a plane.

2. (4 points) Which equations describe a plane? *Circle all that apply.*
   I. $z = 5$
   II. $\rho = \sec\phi$
   III. $ax + by + cz + d = 0$
   IV. $\mathbf{r}(t) = \langle 1+2t, 1+3t, 1-4t\rangle$ $\longleftarrow$ this is a line

3. (4 points) Which equations describe a sphere of radius 3? *Circle all that apply.*
   I. $x^2 + y^2 + z^2 = 9$
   II. $x^2 + y^2 = 9$
   III. $\rho = 3$
   IV. $z = \pm\sqrt{9 - r^2}$.

4. (9 points) In the diagram below, $\mathbf{u}$ and $\mathbf{v}$ form two legs of a triangle. From the list at the right, select the number of the correct expression for the lengths a, b and c pictured.



$a=$ 4

$b=$ 5

$c=$ 1

1. $||\mathbf{u}-\mathbf{v}||$
2. $||\mathbf{v}|| - ||\mathbf{u}||$
3. $\dfrac{|\mathbf{u}\cdot\mathbf{v}|}{||\mathbf{u}||}$
4. $\dfrac{|\mathbf{u}\cdot\mathbf{v}|}{||\mathbf{v}||}$
5. $\dfrac{||\mathbf{u}\times\mathbf{v}||}{||\mathbf{v}||}$

5. (14 points) Given $\mathbf{a} =< 6, -6, 8 >$ and $\mathbf{v} =< 1, -2, 3 >$, find vectors $\mathbf{a}_{||\mathbf{v}}$ and $\mathbf{a}_{\perp\mathbf{v}}$:

(a) $\mathbf{a}_{||\mathbf{v}} =$ $3\vec{v} = \langle 3, -6, 9 \rangle$

$$\vec{a}_{\parallel \vec{v}} = \frac{\vec{a} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v} = \frac{42}{14} \vec{v} = 3\vec{v}$$

(b) $\mathbf{a}_{\perp\mathbf{v}} =$ $\langle 3, 0, -1 \rangle$

$$\vec{a}_{\perp \vec{v}} = \vec{a} - \vec{a}_{\parallel \vec{v}} = \langle 6, -6, 8 \rangle - \langle 3, -6, 9 \rangle$$
$$= \langle 3, 0, -1 \rangle$$

(c) Show that $\mathbf{a}_{\perp\mathbf{v}}$ is perpendicular to $\mathbf{v}$.

$$\vec{a}_{\perp \vec{v}} \cdot \vec{v} = \langle 3, 0, -1 \rangle \cdot \langle 1, -2, 3 \rangle = 0$$

(d) Show that $\mathbf{a}_{||\mathbf{v}}$ is parallel with $\mathbf{v}$.

$\vec{a}_{\parallel \vec{v}} = 3\vec{v}$, clearly parallel.

10 $\leftarrow$ typo

6. ($\cancel{15}$ points) Find an equation for the tangent line to the graph of the curve

$$\mathbf{r}(t) = \left\langle \sqrt{1+3t^2},\ (t+1)\sin 2t,\ e^{4t} \right\rangle$$

at the point $(1, 0, 1)$.

$$\vec{r}\,'(t) = \left\langle \tfrac{1}{2}(1+3t^2)^{-1/2}(6t),\ \sin 2t + 2(t+1)\cos 2t,\ 4e^{4t} \right\rangle$$

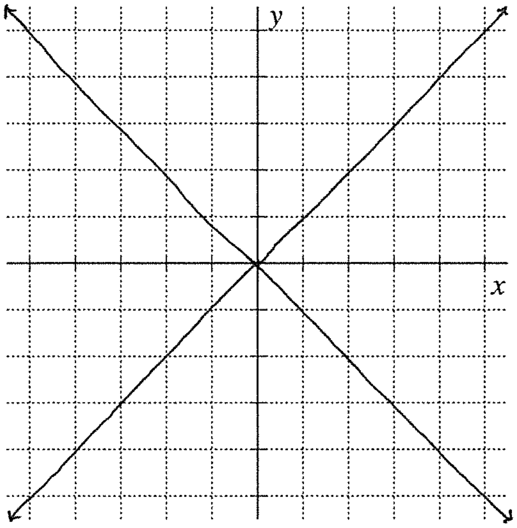
$$\langle 1, 0, 1 \rangle = \vec{r}(0)$$

$$\vec{r}\,'(0) = \langle 0, 2, 4 \rangle$$

so

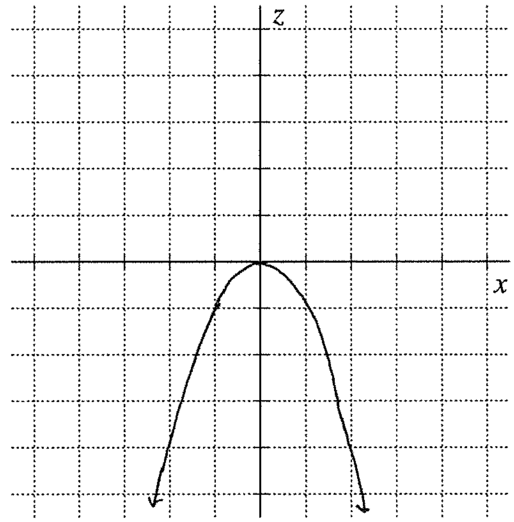
$$\vec{\ell}(t) = \vec{r}(0) + t\,\vec{r}\,'(0)$$
$$= \langle 1, 0, 1 \rangle + t\,\langle 0, 2, 4 \rangle$$

7. (10 points) Provide a clear sketch of the following traces for the quadratic surface $z + x^2 - y^2 = 0$ in the given planes.



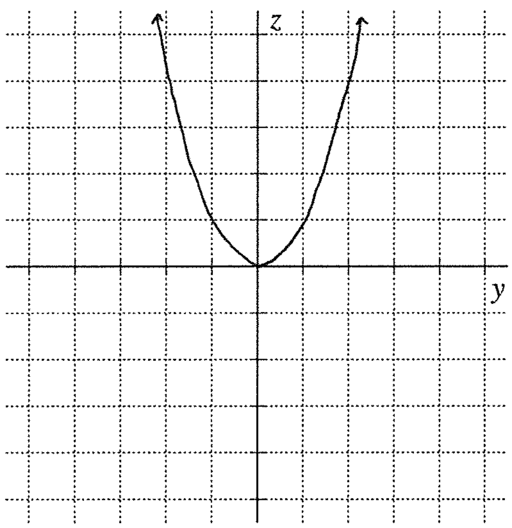
$x^2 = y^2$

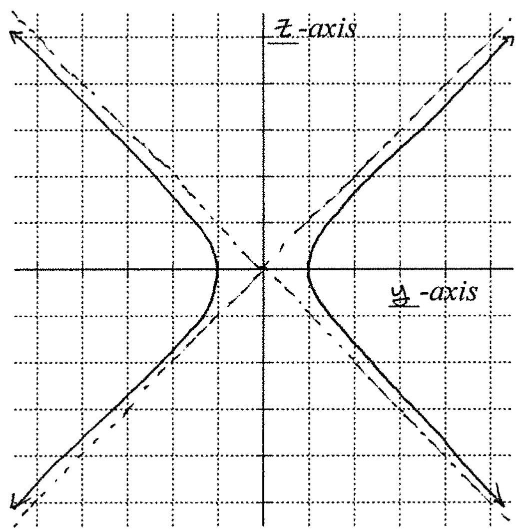
xy–plane



$z = -x^2$

xz–plane





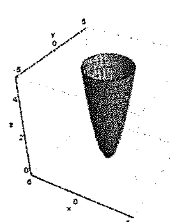
$z = y^2$

yz–plane

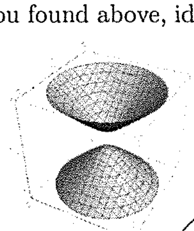
$1 = y^2 - x^2$

$z = 1$ label the appropriate axes.

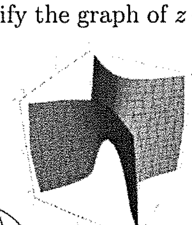
Based on the traces you found above, identify the graph of $z + x^2 - y^2 = 0$ by circling the figure number.



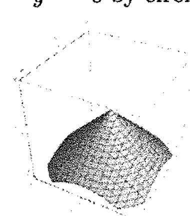
I.



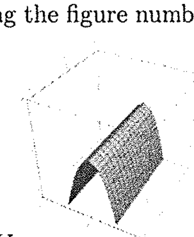
II.



III.



IV.



V.

8. (15 points) Find an equation for the plane that contains the point $(1, 2, 3)$ and the line $x = 1 + t,\ y = -1 + 2t,\ z = -t$.

Let $P = (1,2,3)$

$Q = (1,-1,0) \longleftrightarrow$ the point on the line when $t=0$

$\overrightarrow{PQ} = \langle 0,-3,-3 \rangle$ is a vector in the plane

$\vec{v} = \langle 1, 2, -1 \rangle$ is the direction vector of the line & hence in the plane.

To find a normal we compute

$$\overrightarrow{PQ} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \vec{k} \\ 0 & -3 & -3 \\ 1 & 2 & -1 \end{vmatrix} = \langle 3+6, -(0+3), 3 \rangle = \langle 9, -3, 3 \rangle$$

An equation of the plane is

$$9(x-1) - 3(y-2) + 3(z-3) = 0$$

— OR —

$$3(x-1) - (y-2) + (z-3) = 0$$

— OR —

$$3x - y + z = 4$$

9. (15 points) Find the arc length of the piece of the curve $\mathbf{r}(t) = \langle \cos 3t, \sin 3t, \frac{8}{3}t^{3/2} \rangle$ between the points corresponding to $t = 0$ and $t = 1$.

$$\vec{r}\,'(t) = \langle -3 \sin 3t, 3 \cos 3t, 4 t^{1/2} \rangle$$

$$\| \vec{r}\,'(t) \| = (9 \sin^2 3t + 9 \cos^2 3t + 16t)^{1/2}$$

$$= (9 + 16t)^{1/2}$$

$$S = \int_0^1 (9+16t)^{1/2}\, dt = \frac{1}{16} \cdot \frac{2}{3} (9 + 16t)^{3/2} \Big|_0^1$$

$$= \frac{1}{24} \left(25^{3/2} - 9^{3/2} \right)$$

$$= \frac{1}{24} (125 - 27) = \frac{98}{24} = \frac{49}{12}$$

10. (15 points) Given position $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ at time t, find the following:

a. The unit tangent vector $\mathbf{T}(t) =$

$$\vec{r}\,'(t) = \langle -\sin t, \cos t, 1 \rangle \qquad \|\vec{v}\,'\| = \sqrt{2}$$

$$\vec{T} = \frac{1}{\sqrt{2}} \langle -\sin t, \cos t, 1 \rangle$$

b. The unit normal vector $\mathbf{N}(t) =$

$$\vec{T}\,' = \frac{1}{\sqrt{2}} \langle -\cos t, -\sin t, 0 \rangle \qquad \|\vec{T}\,'\| = \frac{1}{\sqrt{2}}$$

$$\vec{N} = \langle -\cos t, -\sin t, 0 \rangle$$

c. The (scalar) tangential component of acceleration $a_{\mathbf{T}} =$

$$\vec{a} = \langle -\cos t, -\sin t, 0 \rangle$$

$$a_{\vec{T}} = \vec{a} \cdot T = \frac{1}{\sqrt{2}} \left(\sin t \cos t - \cos t \sin t + 0 \right) = 0$$

d. The (scalar) normal component of acceleration $a_{\mathbf{N}} =$

$$a_{\vec{N}} = \vec{a} \cdot \vec{N} = \left(\cos^2 t + \sin^2 t \right) = 1$$

e. The curvature of the graph of $\mathbf{r}(t)$ at $t = \pi/2$, $\kappa(\pi/2) =$ $1/2$.

$$\kappa(t) = \frac{\|\vec{T}\,'\|}{\|\vec{r}\,'\|} = \frac{1/\sqrt{2}}{\sqrt{2}} = \frac{1}{2}$$