**Instructions:** Closed book. No calculator allowed. Double-sided exam. NO CELL PHONES. Show all work and use correct notation to receive full credit! Write legibly.

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\[ \kappa(s) = \left| \frac{dT}{ds} \right| \]
\[ \kappa(x) = \frac{|f''(x)|}{\left[1 + (f'(x))^2\right]^{3/2}} \]
\[ \kappa(t) = \frac{|T'(t)|}{|r'(t)|} \quad a_N = \kappa(t)|v(t)|^2. \]
\[ \kappa(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3} \]

1. (4 points total) True or False? Circle ONE answer for each.  
   (a) True or False: For any vectors \( \mathbf{u}, \mathbf{v} \) in \( \mathbb{R}^3 \), \( \mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u} \). \( \mathbf{u} \times \mathbf{v} = - (\mathbf{v} \times \mathbf{u}) \)
   (b) True or False: \( |r(t)| = 1 \) for all \( t \), then \( |r'(t)| \) is a constant. \( A \) out on a pool bill does not have to have a constant speed
   (c) True or False: The binormal vector is \( \mathbf{B}(t) = \mathbf{N}(t) \times \mathbf{T}(t) \). \( \mathbf{B} = \mathbf{T} \times \mathbf{N} \)
   (d) True or False: A linear equation \( ax + by + cz + d = 0 \) represents a line in space. \( This \ is \ a \ plane. \)

2. (4 points) Which equations describe a plane? **Circle all that apply.**  
   \( I. \ z = 5 \)
   \( II. \ \rho = \sec \phi \)
   \( III. \ ax + by + cz + d = 0 \)
   \( IV. \ r(t) = (1 + 2t, 1 + 3t, 1 - 4t) \)  \( \leftrightarrow \ \mathbf{i} \mathbf{i} \mathbf{i} \mathbf{i} \mathbf{i} \)

3. (4 points) Which equations describe a sphere of radius 3? **Circle all that apply.**  
   \( I. \ x^2 + y^2 + z^2 = 9 \)
   \( II. \ x^2 + y^2 = 9 \)
   \( III. \ \rho = 3 \)
   \( IV. \ r = \pm \sqrt{9 - r^2}. \)
4. (9 points) In the diagram below, \( \mathbf{u} \) and \( \mathbf{v} \) form two legs of a triangle. From the list at the right, select the number of the correct expression for the lengths \( a, b \) and \( c \) pictured.

\[
\begin{align*}
\mathbf{u} & \quad \mathbf{v} \\
a & = 4 \\
b & = 5 \\
c & = 1 \\
\end{align*}
\]

1. \( ||\mathbf{u} - \mathbf{v}|| \)
2. \( ||\mathbf{v}|| - ||\mathbf{u}|| \)
3. \( \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}||} \)
4. \( \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{v}||} \)
5. \( \frac{||\mathbf{u} \times \mathbf{v}||}{||\mathbf{v}||} \)
5. (14 points) Given \( \mathbf{a} = \langle 6, -6, 8 \rangle \) and \( \mathbf{v} = \langle 1, -2, 3 \rangle \), find vectors \( \mathbf{a}_{\parallel \mathbf{v}} \) and \( \mathbf{a}_{\perp \mathbf{v}} \):

(a) \( \mathbf{a}_{\parallel \mathbf{v}} = \frac{3 \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} = \langle 3, -1, 9 \rangle \)

\[
\overrightarrow{\mathbf{a}}_{\parallel \mathbf{v}} = \frac{\mathbf{a} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} \mathbf{v} = \frac{42}{14} \mathbf{v} = 3 \mathbf{v}
\]

(b) \( \mathbf{a}_{\perp \mathbf{v}} = \langle 3, 0, -1 \rangle \)

\[
\overrightarrow{\mathbf{a}}_{\perp \mathbf{v}} = \mathbf{a} - \overrightarrow{\mathbf{a}}_{\parallel \mathbf{v}} = \langle 6, -6, 8 \rangle - \langle 3, -1, 9 \rangle = \langle 3, 5, -1 \rangle
\]

(c) Show that \( \mathbf{a}_{\perp \mathbf{v}} \) is perpendicular to \( \mathbf{v} \).

\[
\overrightarrow{\mathbf{a}}_{\perp \mathbf{v}} \cdot \mathbf{v} = \langle 3, 5, -1 \rangle \cdot \langle 1, -2, 3 \rangle = 0
\]

(d) Show that \( \mathbf{a}_{\parallel \mathbf{v}} \) is parallel with \( \mathbf{v} \).

\[
\overrightarrow{\mathbf{a}}_{\parallel \mathbf{v}} = 3 \mathbf{v}, \quad \mathbf{a}_{\parallel \mathbf{v}} \parallel \mathbf{v}.
\]
6. (25 points) Find an equation for the tangent line to the graph of the curve

\[ r(t) = \left( \sqrt{1 + 3t^2}, (t + 1)\sin(2t), e^{4t} \right) \]

at the point \((1, 0, 1)\).

\[ \vec{r}'(t) = \left( \frac{1}{2} (1 + 3t^2)^{-\frac{1}{2}} (4t), \sin(2t) + 2(t+1) \cos(2t), 4e^{4t} \right) \]

\[ \langle 1, 0, 1 \rangle = \vec{r}'(0) \]

\[ \vec{r}'(0) = \langle 0, 2, 4 \rangle \]

so

\[ \vec{L}(t) = \vec{r}(0) + t \vec{r}'(0) \]

\[ = \langle 1, 0, 1 \rangle + t \langle 0, 2, 4 \rangle \]
7. (10 points) Provide a clear sketch of the following traces for the quadratic surface \( z + x^2 - y^2 = 0 \) in the given planes.

\[ xy\text{-plane} \quad \begin{array}{c}
\text{\( x^2 = y^2 \)}
\end{array} \]

\[ xz\text{-plane} \quad \begin{array}{c}
\text{\( z = -x^2 \)}
\end{array} \]

\[ yz\text{-plane} \quad \begin{array}{c}
\text{\( z = y^2 \)}
\end{array} \]

\[ z = 1 \text{ label the appropriate axes.} \]

Based on the traces you found above, identify the graph of \( z + x^2 - y^2 = 0 \) by circling the figure number.

I.  
II.  
III.  
IV.  
V.
8. (15 points) Find an equation for the plane that contains the point (1, 2, 3) and the line \( x = 1 + t, y = -1 + 2t, z = -t \).

Let \( P = (1, 2, 3) \)

\[ Q = (1, -1, 0) \] is the point on the line when \( t = 0 \)

\( \overrightarrow{PQ} = \langle 0, -3, -3 \rangle \) is a vector in the plane.

\( \overrightarrow{V} = \langle 1, 2, -1 \rangle \) is the direction vector of the line \( \ell \) hence in the plane.

To find a normal we compute

\[
\overrightarrow{PQ} \times \overrightarrow{V} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
0 & -3 & -3 \\
1 & 2 & -1 \\
\end{vmatrix} = \langle 3 + 6, -(0 + 3), 3 \rangle = \langle 9, -3, 3 \rangle
\]

An equation of the plane is

\[ 9(x - 1) - 3(y - 2) + 3(z - 3) = 0 \]

--- or ---

\[ 3(x - 1) - (y - 2) + (z - 3) = 0 \]

--- or ---

\[ 3x - y + z = 4 \]
9. (15 points) Find the arc length of the piece of the curve \( r(t) = \langle \cos 3t, \sin 3t, \frac{8}{3} t^{3/2} \rangle \) between the points corresponding to \( t = 0 \) and \( t = 1 \).

\[
\vec{r}'(t) = \langle -3 \sin 3t, 3 \cos 3t, 4t^{1/2} \rangle
\]

\[
|| \vec{r}'(t) || = \left( 9 \sin^2 3t + 9 \cos^2 3t + 16t \right)^{1/2} = \left( 9 + 16t \right)^{1/2}
\]

\[
S = \int_0^1 \left( 9 + 16t \right)^{1/2} dt = \left. \frac{1}{16} \cdot \frac{2}{3} \left( 9 + 16t \right)^{3/2} \right|_0^1
\]

\[
= \frac{1}{24} \left( 25^{3/2} - 9^{3/2} \right)
\]

\[
= \frac{1}{24} \left( 125 - 27 \right) = \frac{98}{24} = \frac{49}{12}
\]
10. (15 points) Given position \( \mathbf{r}(t) = (\cos t, \sin t, t) \) at time \( t \), find the following:

a. The unit tangent vector \( \mathbf{T}(t) = \) 
\[
\mathbf{T}'(t) = \left< -\sin t, \cos t, 1 \right> \quad \| \mathbf{T}' \| = \frac{1}{\sqrt{2}}
\]

\[
\mathbf{T} = \frac{1}{\sqrt{2}} \left< -\sin t, \cos t, 1 \right>
\]

b. The unit normal vector \( \mathbf{N}(t) = \) 
\[
\mathbf{N}'(t) = \frac{1}{\sqrt{2}} \left< -\cos t, -\sin t, 0 \right> \quad \| \mathbf{N}' \| = \frac{1}{\sqrt{2}}
\]

\[
\mathbf{N} = \left< -\cos t, -\sin t, 0 \right>
\]

c. The (scalar) tangential component of acceleration \( a_T = \) 
\[
\mathbf{a} = \left< -\cos t, -\sin t, 0 \right>
\]

\[
\mathbf{a} \cdot \mathbf{T} = \frac{1}{\sqrt{2}} \left( \sin t \cos t - \cos t \sin t + 0 \right) = 0
\]

d. The (scalar) normal component of acceleration \( a_N = \) 
\[
a_N = \mathbf{a} \cdot \mathbf{N} = \left( \omega^2 t + \sin^2 t \right) \quad \|
\]

e. The curvature of the graph of \( \mathbf{r}(t) \) at \( t = \pi/2 \), \( \kappa(\pi/2) = \) 
\[
\kappa(\tau) = \frac{\| \mathbf{T}' \|}{\| \mathbf{T} \|^2} = \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{2}
\]