M273Q, FALL 2016 Name:______________________________

Section: __________

EXAM 2; October 24, 2016; 100 points

Instructions: Closed book. No calculator allowed. Double-sided exam. NO CELL PHONES. Show all work and use correct notation to receive full credit! Write legibly.

Problem	1	2	3	4	5	6	7	8	9	Total
Points Possible	27	10	8	10	8	3	4	15	15	100
Points Earned										

1. Let $g(x,y) = e^{x^2-y^3}$. Note that $\nabla g(x,y) = \left\langle 2xe^{x^2-y^3}, -3y^2e^{x^2-y^3}\right\rangle$.

   (a) (5 points) Find the maximum rate of change of g at $(1,1)$.

   $\nabla g(1,1) = \langle 2,-3\rangle$ so max R.o.C. is $\|\nabla g(1,1)\| = \sqrt{13}$

   (b) (5 points) Find the rate of change of g at $(1,1)$ in the direction of $<2,1>$.

   $D_{\langle 2,1\rangle} g(1,1) = \nabla g(1,1)\cdot \frac{\langle 2,1\rangle}{\sqrt{5}} = \frac{1}{\sqrt{5}}$

   (c) (5 points) Find a direction in which the rate of change of g at $(1,1)$ is zero.

   We need a vector so that $\nabla g(1,1)\cdot\vec{v} = 0$,

   one such vector is $\langle 3,2\rangle$.

   (d) (8 points) Find an equation for the tangent plane to the surface $g(x,y) = e^{x^2-y^3}$ at the point $(1,1)$.

   $z = g(1,1) + g_x(1,1)(x-1) + g_y(1,1)(y-1)$

   $= 1 + 2(x-1) - 3(y-1)$

   (e) (4 points) Use the linear approximation to find an approximate value for $z = g(x,y)$ when $x = 1.1$ and $y = 0.9$.

   $g(1.1, 0.9) \approx 1 + 2(1.1-1) - 3(0.9-1) = 1.5$

2. (10 points) Calculate the partial derivative g_{zzwx} where $g(x,y,z,w) = x^3w^2z^2 + \sin\left(\frac{xy}{z^2}\right)$.

By Clairaut's Theorem

$g_{zzwx} = g_{wxzz}$

$g_w = 2x^3wz^2$

$g_{wx} = 6x^2wz^2$

$g_{wxz} = 12x^2wz$

$g_{wxzz} = 12x^2w$

3. (8 points) Given $f(x,y) = (x^2+y)e^{(x-y)}$, find the gradient of f.

$\nabla f(x,y) = \langle f_x(x,y),\ f_y(x,y) \rangle$

$= \langle 2xe^{x-y} + (x^2+y)e^{x-y},\ e^{x-y} - (x^2+y)e^{x-y} \rangle$

4. (10 points) Let

$$u(x,y,z) = \frac{xy^2}{z^3}, \quad x(p,r,t) = 3p + 6r^{16} + 5\ln t, \quad y(p,r,t) = 5p - 6\ln r + t^{\sin(t)}, \quad z(p,r,t) = 7p + 6r^4 - \frac{1}{t}.$$

Find $\dfrac{\partial u}{\partial p}$.

$$\frac{\partial u}{\partial p} = \frac{\partial u}{\partial x}\,\frac{\partial x}{\partial p} + \frac{\partial u}{\partial y}\,\frac{\partial y}{\partial p} + \frac{\partial u}{\partial z}\,\frac{\partial z}{\partial p}$$

$$= \frac{y^2}{z^3}(3) + \frac{2xy}{z^3}(5) - 3\,\frac{xy^2}{z^4}(7)$$

5. (8 points total) Show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^2}$$

Consider the limit in polar where $2xy = 2(r\cos\theta)(r\sin\theta)$

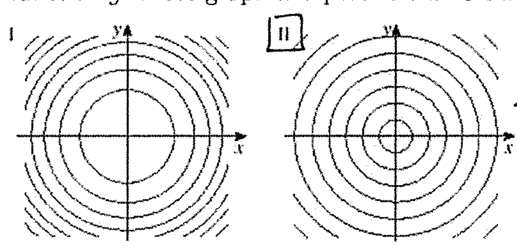
and $x^2+2y^2 = x^2+y^2+y^2 = r^2 + r^2\sin^2\theta$

so $$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^2} = \lim_{r\to 0^+} \frac{2r^2\cos\theta\sin\theta}{r^2(1+\sin^2\theta)} = \lim_{r\to 0^+} \frac{2\cos\theta\sin\theta}{1+\sin^2\theta}$$

$$= \frac{2\cos\theta\sin\theta}{1+\sin^2\theta}.$$

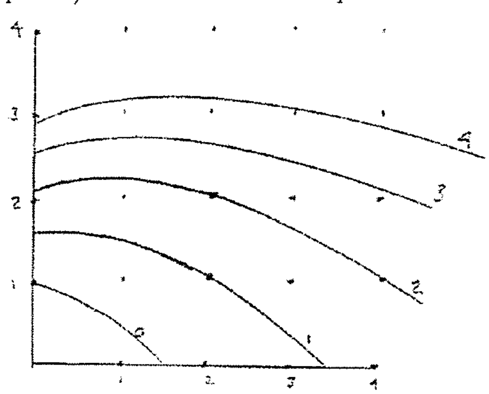
Since the limit depends on θ, $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^2}$ does not exist

6. (3 points) Two contour maps are shown. One is for a function f whose graph is a cone. The other is for a function g whose graph is a paraboloid. *Clearly* identify which contour map is that of a **cone.**



$\leftarrow$ CONE

7. (4 points) Below is a level curve picture for the function $f(x,y)$.



Circle the best answer:

(a) $f_x(2,1)$ equals i. -2 $\boxed{\text{ii. } 1/2}$ iii. 1 iv. 2

(b) $f_y(2,1)$ equals i. -1 ii. 0 iii. $1/2$ $\boxed{\text{iv. } 1}$

8. (15 points) Find all critical points of $f(x,y) = x^2 + 3xy + y^2 - 4x - y$ and classify them (local maximum, local minimum, or saddle) using the Second Derivative Test.

$f_x = 2x + 3y - 4$ so $f_{xx} = 2$

$f_{xy} = f_{yx} = 3$

$f_y = 3x + 2y - 1$ so $f_{yy} = 2$

$f_x = 0$ when $2x + 3y = 4$ (1)

$f_y = 0$ when $3x + 2y = 1$ (2)

multiplying (1) by 3 & subtracting (2) times 2 gives

$$\begin{array}{r} 6x + 9y = 12 \\ 6x + 4y = 2 \\ \hline 5y = 10 \end{array}$$

so $y = 2$ so $x = -1$

$(-1, 2)$ is the only critical point.

Since $D = f_{xx} f_{yy} - (f_{xy})^2 = 4 - 9 = -5 < 0$,

it is a saddle point.

9. (15 points) Find the maximum and minimum of $f(x, y) = xy$ on the ellipse $x^2 + 2y^2 = 8$.

Using the method of Lagrange we have

$$\nabla f = \lambda \nabla g \quad \text{where} \quad g(x,y) = \underbrace{x^2 + 2y^2 = 8}_{} \qquad (1)$$

$$y = \lambda (2x) \qquad (2)$$

$$x = \lambda (4y) \qquad (3)$$

Solving (2) for λ gives $\lambda = \dfrac{y}{2x}$

(or $x = y = 0$ which is not a valid point on the ellipse)

Solving (3) for λ gives $\lambda = \dfrac{x}{4y}$ (or $x = y = 0$ as above)

Equating gives $\dfrac{y}{2x} = \dfrac{x}{4y}$ so $2y^2 = x^2$

Substituting into (1) gives $x^2 + x^2 = 8$ or $x^2 = 4$ or $x = \pm 2$

so $y = \pm\sqrt{2}$

Max

$$f(2, \sqrt{2}) = 2\sqrt{2}$$

$$f(-2, \sqrt{2}) = -2\sqrt{2}$$

$$f(2, -\sqrt{2}) = -2\sqrt{2}$$

$$f(-2, -\sqrt{2}) = 2\sqrt{2}$$

Min