Math 273 — Divergence Thing — Name: ________________

8 Dec 2016 — Show appropriate work. — Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ Let $\mathcal{W}$ be the solid sphere of radius 2 centered at the origin with positively oriented boundary $\mathcal{S} = \partial\mathcal{W}$, i.e. outward normal vectors. Let $\mathbf{F} = \langle y^{2016}, \sin z^{2016}, 2016^x \rangle$, compute
$$\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}.$$
$$= \iiint_{\omega} (0)\, dV = 0$$

2. $\boxed{2}$ Let $\mathcal{W}$ be the solid sphere of radius 2 centered at the origin with negatively oriented boundary $\mathcal{S} = \partial\mathcal{W}$, i.e. inward normal vectors. Let $\mathbf{F} = \langle x + y^{2016}, y + \sin z^{2016}, z + 2016^x \rangle$, compute
$$\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}.$$
$$= -\iiint_{\omega} (3)\, dV = -3 \text{ volume } (\omega) = -3 \cdot \frac{4}{3}\pi (2)^3 = -32\pi$$

3. $\boxed{6}$ Let $\mathcal{W}$ be the solid sphere of radius 2 centered at the origin with positively oriented boundary $\mathcal{S} = \partial\mathcal{W}$, i.e. outward normal vectors. Let $\mathbf{F} = \langle 3x + y^{2016}, \sin z^{2016}, z^4 + 2016^x \rangle$, compute
$$\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}.$$
$$= \iiint_{\omega} (3 + 4z^3)\, dV = \int_0^{2\pi}\int_0^{2}\int_0^{\pi} (3 + 4\cos^3\phi\, \rho^3)\, \rho^2 \sin\phi \, d\phi\, d\rho\, d\theta$$
$$= 2\pi \int_0^2 \left[-3\rho^2 \cos\phi - \rho^5 \cos^4\phi \right]\Big|_0^{\pi} d\rho$$
$$= 2\pi \int_0^2 \left[-3\rho^2(-2) \right] d\rho = 4\pi \rho^3 \Big|_0^2 = 32\pi$$