Math 273 | Quiz 4 | Name: _____

23 Sept 2016 | Show appropriate work. | Point Values in boxes.

1. [2] Please indicate **T**rue or **F**alse.
   (a) (T) / **F** : $\mathbf{T} = \mathbf{v}/v$.
   (b) **T** / (F) : $\mathbf{N} = \mathbf{a}/\|\mathbf{a}\|$.
   (c) (T) / **F** : If the speed is constant, then $\mathbf{a} = a_{\mathbf{N}}\mathbf{N}$.
   (d) **T** / (F) : If the curvature is constant, then $\mathbf{a} = a_{\mathbf{T}}\mathbf{T}$.

Notation!

2. Consider the path given by $\mathbf{r}(t) = \langle t - t^3/3, t^2 \rangle$.

   (a) [1] Find the velocity $\mathbf{v}(t)$.

   $\vec{r}\,' = \langle 1 - t^2, 2t \rangle$

   (b) [1] Find the speed $v(t)$.

   $\|\vec{r}\,'\| = (1 - 2t^2 + t^4 + 4t^2)^{1/2} = (1 + 2t^2 + t^4)^{1/2} = (1+t^2)$

   (c) [1] Find the acceleration $\mathbf{a}(t)$.

   $\vec{r}\,'' = \langle -2t, 2 \rangle$

   (d) [1] Find the Unit tangent vector $\mathbf{T}(t)$.

   $\left\langle \frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2} \right\rangle$

   (e) [4] For this path the Unit normal vector is $\mathbf{N}(t) = \left\langle \frac{-2t}{1+t^2}, \frac{1-t^2}{1+t^2} \right\rangle$. Using this and the above, find the tangential and normal components of acceleration, ie. $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$, at $t = 2$.

$\vec{a}(2) = \langle -4, 2 \rangle$

$\vec{N}(2) = \left\langle \frac{-4}{5}, \frac{-3}{5} \right\rangle$ $\qquad$ $\vec{T}(2) = \left\langle \frac{-3}{5}, \frac{4}{5} \right\rangle$

$a_{\vec{T}} = \vec{a} \cdot \vec{T} = \frac{12}{5} + \frac{8}{5} = 4$

$a_N = \vec{a} \cdot \vec{N} = \frac{16}{5} - \frac{6}{5} = 2$

OR $a_{\vec{T}} = v'(2) = 2t \Big|_{t=2} = 4$

$\because \|\vec{a}\|^2 = a_{\vec{T}}^2 + a_{\vec{N}}^2$

so $a_{\vec{N}} = 2$

Note: $v(2) = 5$ $\quad v^2 K = 2$ $\quad 25K = 2$, $K = \frac{2}{25}$