Math 273 | Quiz 5 | Name: ____________
7 Oct 2016 | Show appropriate work. | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{4}$ Show the limit does not exist. [HINT: Consider two paths, lines will do.]

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

Along the x-axis, i.e. $y=0$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{0}{x^2} = 0.$$

Along the line $y=x$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+x^2} = \frac{1}{2}.$$

Since the limits differ,

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} \text{ does not exist.}$$

Along the path $y=mx$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{mx^2}{x^2+m^2x^2} = \frac{m}{1+m^2}.$$

OR Since the limit depends on m,

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} \text{ does not exist.}$$

2. $\boxed{2}$ Compute f_{xyy} for $f(x,y) = x^3y^2 + y\sin\left(\frac{x+x^3}{1+x^4}\right)$.

By Clairaut's Theorem, $f_{xyy} = f_{yyx}$.

$$f_y = 2x^3y + \sin\left(\frac{x+x^3}{1+x^4}\right)$$

$$f_{yy} = 2x^3$$

$$f_{yyx} = 6x^2$$

3. $\boxed{4}$ Find an equation for the tangent plane to $f(x,y) = xe^x + y^2$ at $(0,1)$.

$f_x = e^x + xe^x$ $\quad$ $f_x(0,1) = 1$ $\quad$ $f(0,1) = 1$

$f_y = 2y$ $\quad$ $f_y(0,1) = 2$

so

$z = 1 + 1(x-0) + 2(y-1)$ is the tangent plane

—— OR ——

$z = 1 + x + 2y - 2$ / OR / $z = x + 2y - 1$