Math 273 | Quiz 6 | Name: ____________________

14 Oct 2016 | Show appropriate work. | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ Please indicate **T**rue or **F**alse.

   (a) **T** / **F** : $\nabla f(x,y)$ is normal to the surface $z = f(x,y)$.

   (b) **T** / **F** : If $\nabla f(a,b) = \mathbf{0}$, then (a,b) is a critical point of $f(x,y)$.

   (c) **T** / **F** : A continuous function defined on a bounded domain $\mathscr{D}$ has an absolute maximum.

   (d) **T** / **F** : If a function has a absolute maximum on a domain $\mathscr{D}$, it occurs at a critical point.

2. Consider the function $f(x,y) = 3x^2y + y^3 - 3y$.

   (a) $\boxed{2}$ Find $\nabla f(x,y)$.

   $\nabla f(x,y) = \langle 6xy,\ 3x^2 + 3y^2 - 3 \rangle$

   (b) $\boxed{2}$ Find the directional derivative in the direction of $\mathbf{v} = \langle 3, 4\rangle$ at $(1,1)$.

   $D_{\vec{v}}(1,1) = \nabla f(1,1) \cdot \dfrac{\langle 3,4\rangle}{\sqrt{9+16}} = \langle 6, 3\rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle = \frac{30}{5} = 6$

   (c) $\boxed{2}$ Find all critical points.

   $f_x = 6xy = 0$ when $x = 0$ or $y = 0$

   if $x = 0$, $f_y = 3(y^2 - 1) = 0$ when $y = \pm 1$

   if $y = 0$, $f_y = 3(x^2 - 1) = 0$ when $x = \pm 1$

   so $(\pm 1, 0)$ & $(0, \pm 1)$ are critical points

   (d) $\boxed{2}$ Use the Second Derivative Test to classify each critical point.

   $D = f_{xx} f_{yy} - (f_{xy})^2 = (6y)(6y) - (6x)^2 = 36(y^2 - x^2)$

C.P	D	f_{xx}	Type
(1,0)	-36		Saddle
(-1,0)	-36		—"—
(0,1)	36	6	Local Min
(0,-1)	36	-6	Local Max