Math 273 | Quiz 7 | Name: ____________________
28 Oct 2016 | Show appropriate work. | Point Values in $\boxed{\text{boxes}}$ - Scaled to 10.

Choose three out of four.

1. $\boxed{10}$ Integrate.

$$\int_1^2 \int_{1/x}^1 \frac{x}{1+xy}\, dy\,dx$$

$$= \int_1^2 \ln(1+xy)\Big|_{1/x}^{1} dx = \int_1^2 [\ln(1+x) - \ln(2)]\, dx$$

$u = \ln(1+x) - \ln 2 \qquad dv = dx$

$du = \frac{1}{1+x}\,dx \qquad v = x$

$$= x(\ln(1+x) - \ln 2)\Big|_1^2 - \int_1^2 \frac{x}{1+x}\,dx \quad \leftarrow \text{Note: } \frac{x}{1+x} = \frac{1+x}{1+x} - \frac{1}{1+x}$$

$$= 2(\ln 3 - \ln 2) - [x - \ln(1+x)]\Big|_1^2$$

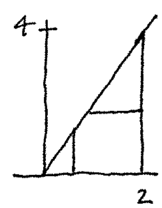
$$= 2(\ln 3 - \ln 2) - [2 - \ln 3 - 1 + \ln 2]$$

$$= 3\ln 3 - 3\ln 2 - 1$$

2. $\boxed{10}$ Change the order of integration and then evaluate.

$x = \frac{y}{2}$ or $y = 2x$

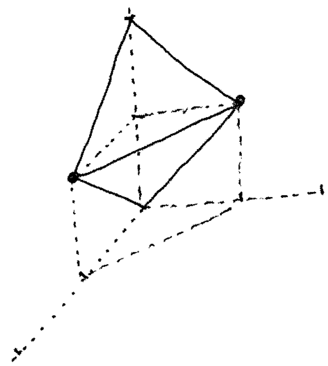
$$\int_0^4 \int_{y/2}^2 e^{x^2}\, dx\,dy$$



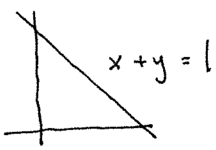
$$= \int_0^2 \int_0^{2x} e^{x^2}\, dy\, dx$$

$$= \int_0^2 2x\, e^{x^2}\, dx = e^{x^2}\Big|_0^2 = e^4 - 1$$

3. $\boxed{10}$ Express the volume in the first octant (i.e. $x, y, z \geq 0$) bounded between the planes $x+y-z=0$ and $x+y+z=2$ as a triple integral and then evaluate it.



proj in xy-plane



$$\int_0^1\int_0^{1-x}\int_{x+y}^{2-x-y} dz\,dy\,dx = \int_0^1\int_0^{1-x}(2-x-y-x-y)\,dy\,dx$$

$$= \int_0^1 (2-2x)y - y^2\Big|_0^{1-x} dx = \int_0^1 \left[2(1-x)^2-(1-x)^2\right]dx$$

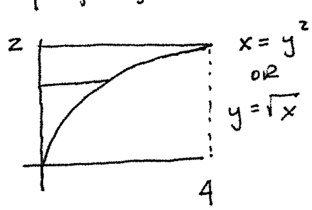
$$= \int_0^1 (1-x)^2\,dx = -\frac{(1-x)^3}{3}\Big|_0^1 = \frac{1}{3}$$

4. Consider the triple integral

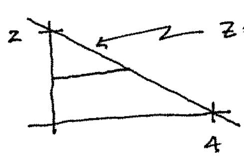
$$\int_0^2\int_0^{y^2}\int_0^{2-x/2} f(x,y,z)\,dz\,dx\,dy.$$

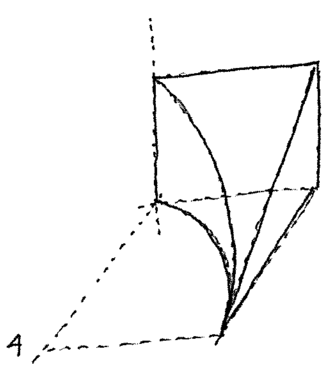
(a) $\boxed{8}$ Convert to an integral in $dydxdz$ order.

proj - xy



proj - xz





$$\int_0^2\int_0^{4-2z}\int_{\sqrt{x}}^{2} f(x,y,z)\,dy\,dx\,dz$$

(b) $\boxed{2}$ In which, if any, orders would this require two or more integrals.

dx dy dz AND dx dz dy