Math 273 | Quiz 8 | Name: ______________
4 Nov 2016 | Show appropriate work. | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{6}$ Let $\mathcal{W}$ be the part of the unit sphere $\rho \le 1$ in the first octant, i.e. $x, y, z \ge 0$. Use spherical coordinates to evaluate

$$\iiint_{\mathcal{W}} xy\, dV.$$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{1} (\rho \sin\phi \cos\theta)(\rho \sin\phi \sin\theta)\, \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \, d\theta \int_0^{\pi/2} \sin^3\phi \, d\phi \int_0^1 \rho^4 \, d\rho = \frac{\sin^2\theta}{2}\Bigg|_0^{\pi/2} \int_0^{\pi/2} (1-\cos^2\phi)\sin\phi\, d\phi \left(\frac{\rho^2}{5}\Bigg|_0^1 \right)$$

$$= \frac{1}{10}\left(\frac{\cos^3\phi}{3} - \cos\phi \right)\Bigg|_0^{\pi/2} = \frac{1}{10}\left(\frac{2}{3}\right) = \frac{1}{15}$$

2. Let $\mathbf{F}(x, y, z) = \langle xy, xyz, ye^y \rangle$. Compute the following.

(a) $\boxed{2}$ $\mathrm{div}(\mathbf{F}) = \nabla \cdot \vec{F} = y + xz$

(b) $\boxed{2}$ $\mathrm{curl}(\mathbf{F}) = \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & xyz & ye^y \end{vmatrix} = \langle e^y + ye^y - xy,\ 0,\ yz - x \rangle$