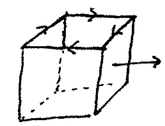
Math 273 | Stokes' Thing | Name: ______________

5 Dec 2016 | Show appropriate work. | Point Values in $\boxed{\text{boxes}}$.

1. Let S be the surface of the box with sides of length 2 with opposite verticies at $(0,0,0)$ and $(2,2,2)$ and no 'top'. Normal vectors point outward. Evaluate



$$\iint_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$$

where $\mathbf{F} = \left\langle -xy, xy, xy^2 z^{2016} + 2016^{x^2+y^2+z^2} \right\rangle$.

(a) $\boxed{5}$ Use Stokes' Theorem to evaluate the integral as a line integral around the boundary.

$\vec{C}_1 = \langle 0, 2t, 2 \rangle$ $\quad \vec{C}_1' = \langle 0, 2, 0 \rangle$ $\quad \vec{F}(\vec{C}_1) = \langle 0, 0, — \rangle$ $\quad \vec{F} \cdot \vec{C}_1' = 0$

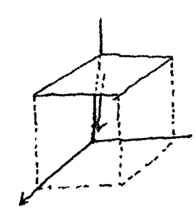
$\vec{C}_2 = \langle 2t, 2, 2 \rangle$ $\quad \vec{C}_2' = \langle 2, 0, 0 \rangle$ $\quad \vec{F}(\vec{C}_2) = \langle -4t, —, — \rangle$ $\quad \vec{F} \cdot \vec{C}_2' = -8t$

$\vec{C}_3 = \langle 2, 2-2t, 2 \rangle$ $\quad \vec{C}_3' = \langle 0, -2, 0 \rangle$ $\quad \vec{F}(\vec{C}_3) = \langle —, 4-4t, — \rangle$ $\quad \vec{F} \cdot \vec{C}_3' = 8t - 8$

$\vec{C}_4 = \langle 2-2t, 0, 2 \rangle$ $\quad \vec{C}_4' = \langle -2, 0, 0 \rangle$ $\quad \vec{F}(\vec{C}_4) = \langle 0, —, — \rangle$ $\quad \vec{F} \cdot \vec{C}_4' = 0$

$$\int_{\partial S} \vec{F} \cdot d\vec{r} = \int_0^1 (-8t)\,dt + \int_0^1 (8t-8)\,dt = \int_0^1 (-8)\,dt = -8$$

(b) $\boxed{5}$ Use Stokes' Theorem to evaluate the integral as a surface integral over the 'top,' i.e. deform the surface into an easier one with the same boundary.



$G(x, y, 2)$ $\quad 0 \le x, y \le 2$ is a parameterization

$\vec{T}_x = \langle 1, 0, 0 \rangle$

$\vec{T}_y = \langle 0, 1, 0 \rangle$

$\vec{N} = \langle 0, 0, -1 \rangle$

$\operatorname{curl}(\vec{F}) = \langle \text{stuff}, \text{messy stuff}, y + x \rangle$

$\operatorname{curl}(\vec{F}) \cdot \vec{N} = (-x - y)$

$$\iint_{S^*} \operatorname{curl}(\vec{F}) \cdot d\vec{S} = \int_0^2 \int_0^2 (-x-y)\,dy\,dx = \int_0^2 \left(-xy - \frac{y^2}{2}\right)\Big|_0^2 dx = \int_0^2 (-2x-2)\,dx = -x^2 - 2x \Big|_0^2 = -8$$