Math 273 Surface Thing Name: ________________

Due: 2 Dec 2016 Show appropriate work. Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{5}$ Evaluate

$$\iint_S \langle yz, xz, x^2+y^2\rangle \cdot d\mathbf{S}$$

where S is the cone parameterized by

$$G(u,v) = (2u\cos v, 2u\sin v, 3u)$$

for $0 \le u \le 2$ and $0 \le v \le 2\pi$, with outward/downward pointing normal vectors.

$$\vec{T}_u = \langle 2\cos v,\ 2\sin v,\ 3\rangle$$

$$\vec{T}_v = \langle -2u\sin v,\ 2u\cos v,\ 0\rangle$$

$$\vec{T}_u \times \vec{T}_v = \langle -6u\cos v,\ -6u\sin v,\ 4u\rangle \longleftarrow \text{upward}$$

$$\vec{N} = \langle 6u\cos v,\ 6u\sin v,\ -4u\rangle \text{ is downward}$$

$$\vec{F} = \langle 6u^2\sin v,\ 6u^2\cos v,\ 4u^2\rangle$$

$$\vec{F}\cdot\vec{N} = 36u^3\cos v\sin v + 36u^3\cos v\sin v - 16u^3 = 72u^3\cos v\sin v - 16u^3$$

$$\iint_S \vec{F}\cdot d\vec{S} = \int_0^2\int_0^{2\pi}(72u^3\cos v\sin v - 16u^3)\,dv\,du = \int_0^2 \left(72u^3\frac{\sin^2 v}{2} - 16u^3 v\right)\Big|_0^{2\pi} du$$

$$= -32\pi\int_0^2 u^3\,du = -32\pi\left(\frac{u^4}{4}\right)\Big|_0^2 = -128\pi$$

2. $\boxed{5}$ Evaluate

$$\iint_S \langle xz, yz, x^2+y^2\rangle \cdot d\mathbf{S}$$

where S is the surface consisting of the cylinder $x^2+y^2=4$ for $0 \le z \le 3$ with outward pointing normal and the disk $x^2+y^2 \le 4$ in the xy-plane with downward normal. [S is a drinking glass, with normal vectors pointing outward.]

S_1: 

$G(\theta, z) = \langle 2\cos\theta, 2\sin\theta, z\rangle \qquad 0 \le \theta \le 2\pi, \quad 0 \le z \le 3$

$\vec{N} = \langle 2\cos\theta, 2\sin\theta, 0\rangle \leftarrow$ outward

$\vec{F} = \langle 2z\cos\theta, 2z\sin\theta, 4\rangle$

$\vec{F}\cdot\vec{N} = 4z\cos^2\theta + 4z\sin^2\theta = 4z$

$$\iint_{S_1} \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_0^3 4z \, dz\, d\theta = 2\pi\left(2z^2\right)\Big|_0^3 = 36\pi$$

S_2: 

$G(\theta, r) = \langle r\cos\theta, r\sin\theta, 0\rangle \qquad 0 \le r \le 2, \quad 0 \le \theta \le 2\pi$

$\vec{N} = \langle 0, 0, -r\rangle$ is downward

$\vec{F} = \langle 0, 0, r^2\rangle$

$\vec{F}\cdot\vec{N} = -r^3$

$$\iint_{S_2} \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_0^2 (-r^3)\, dr\, d\theta = 2\pi\left(\frac{-r^4}{4}\right)\Big|_0^2 = -8\pi$$

So $$\iint_S \vec{F}\cdot d\vec{S} = 36\pi - 8\pi = 28\pi$$