

1. 1 Proper notation.
2. 3 Find an equation for the plane containing the point  $(1, 2, 3)$  and the line  $\mathbf{r}(t) = \langle 1 + 2t, t, 2 - t \rangle$ .

Let  $P = (1, 2, 3)$   
 $Q = (1, 0, 2)$   
 $\overrightarrow{PQ} = \langle 0, -2, -1 \rangle$   
 $\vec{v} = \langle 2, 1, -1 \rangle$

$$\overrightarrow{PQ} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & -1 \\ 2 & 1 & -1 \end{vmatrix} = \langle 3, -2, 4 \rangle$$

so an equation is

$$3(x-1) - 2(y-2) + 4(z-3) = 0$$

or

$$3x - 2y + 4z = 11$$

3. 3 Find an equation for the plane containing the lines  $\mathbf{r}(t) = \langle 1, 2 + t, 3 - 2t \rangle$  and  $\mathbf{s}(t) = \langle 1 - t, 2 + 2t, 3 + t \rangle$ .

$(1, 2, 3)$  is a point on both lines.

The direction vectors of the lines are  $\vec{u} = \langle 0, 1, -2 \rangle$  &  $\vec{v} = \langle -1, 2, 1 \rangle$ .

A normal vector is then  $\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -2 \\ -1 & 2 & 1 \end{vmatrix} = \langle 5, 2, 1 \rangle$ .

so an equation is

$$5(x-1) + 2(y-2) + (z-3) = 0$$

or

$$5x + 2y + z = 12$$

4. 3 Find an equation for the tangent plane to  $f(x, y) = \frac{x^2 y^2}{x^2 + y^2}$  at  $(1, 2)$ .

$$f_x = \frac{(x^2 + y^2)(2xy^2) - x^2 y^2(2x)}{(x^2 + y^2)^2}$$

$$f_x(1, 2) = \frac{5 \cdot 8 - 8}{5^2} = \frac{32}{25}$$

$$f_y = \frac{(x^2 + y^2)(2x^2 y) - x^2 y^2(2y)}{(x^2 + y^2)^2}$$

$$f_y(1, 2) = \frac{5 \cdot 4 - 16}{5^2} = \frac{4}{25}$$

$$f(1, 2) = \frac{4}{5}$$

so an equation is

$$z = \frac{4}{5} + \frac{32}{25}(x-1) + \frac{4}{25}(y-2)$$