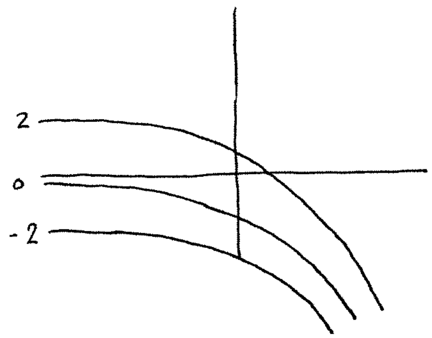
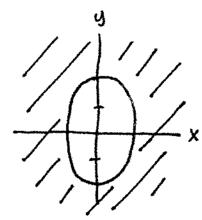
HW 2

1. $c = e^x + y$, $c = -2, 0, 2$ i.e. $y = c - e^x$

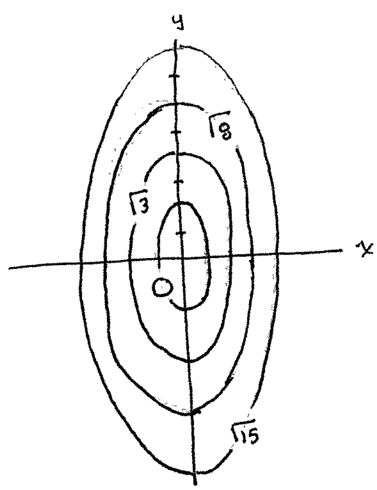


2. $f(x,y) = \sqrt{x^2 + \left(\frac{y}{2}\right)^2 - 1}$

A. Domain $D = \left\{ (x,y) \mid x^2 + \left(\frac{y}{2}\right)^2 \geq 1 \right\}$



B. $c = \sqrt{x^2 + \left(\frac{y}{2}\right)^2 - 1}$, $c = 0, \sqrt{3}, \sqrt{8}, \sqrt{15}$

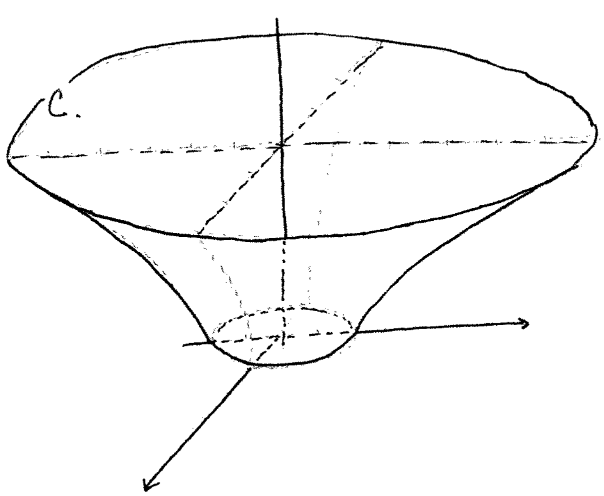


$c = 0$, $1 = x^2 + \left(\frac{y}{2}\right)^2$

$c = \sqrt{3}$, $4 = x^2 + \left(\frac{y}{2}\right)^2$

$c = \sqrt{8}$, $9 = x^2 + \left(\frac{y}{2}\right)^2$

C.



Top half of a hyperboloid of one sheet.

3. A. $\displaystyle\lim_{(x,y)\to(2,\pi)} \frac{x\cos y}{x^2+\sin^2 y} = \frac{2(-1)}{4+0} = -\frac{1}{2}$

B. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = \lim_{r\to 0^+} \frac{\sin(r^2)}{r^2} = 1$

C. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x\arcsin y}{x^2+y^2}$

Along the path $x=0$ $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x\arcsin y}{x^2+y^2} = 0$

Along the path $y=\sin x$ $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x\arcsin y}{x^2+y^2} = \lim_{(x,\sin x)\to(0,0)} \frac{x^2}{x^2+\sin^2 x} = \lim_{(x,\sin x)\to(0,0)} \frac{1}{1+\left(\frac{\sin x}{x}\right)^2} = \frac{1}{2}$

Since the limits along two different paths are different,

$\displaystyle\lim_{(x,y)\to(0,0)} \frac{x\arcsin y}{x^2+y^2}$ does not exist.

4. C. $\displaystyle f_y(x,y) = \frac{(x^3-3xy^2)(x^2+y^2)-(x^3y-xy^3)(2y)}{(x^2+y^2)^2} = \frac{x(x^4-4x^2y^2-y^4)}{(x^2+y^2)^2}$ for $(x,y)\neq(0,0)$

E. $\displaystyle f_y(0,0) = \lim_{h\to 0} \frac{f(0,h)-f(0,0)}{h} = 0$

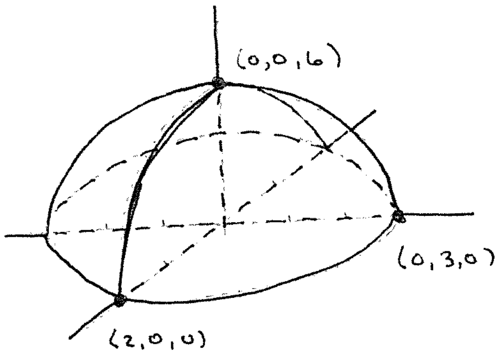
F. $\displaystyle f_{xy}(0,0) = \lim_{h\to 0} \frac{f_x(0,h)-f_x(0,0)}{h} = \lim_{h\to 0} \frac{-h-0}{h} = -1$

$\displaystyle f_{yx}(0,0) = \lim_{h\to 0} \frac{f_y(h,0)-f_y(0,0)}{h} = \lim_{h\to 0} \frac{h-0}{h} = 1$

G. Since $f_{xy}(0,0)\neq f_{yx}(0,0)$, One or both of f_{xy} & f_{yx} is not continuous at $(0,0)$.

5. $f(x,y) = \sqrt{36 - 9x^2 - 4y^2}$

A.



B. $df = \dfrac{-9x}{\sqrt{36-9x^2-4y^2}}\,dx - \dfrac{4y}{\sqrt{36-9x^2-4y}}\,dy$

C. $\Delta f = \cancel{f(1.01) - f}$
$f(1.01, 1.97) - f(1,2) \approx 0.044$

$df = \dfrac{-9}{\sqrt{11}}(0.01) - \dfrac{8}{\sqrt{11}}(-0.03) \approx 0.045$

D. $z - \sqrt{11} = \dfrac{-9}{\sqrt{11}}(x-1) - \dfrac{8}{\sqrt{11}}(y-2)$

E. $z = 6$