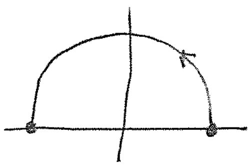
Math 273 | Quiz 10 | Name: ______________

15 April 2016 | Show appropriate work. | Scaled to 10 points.

1. $\boxed{5}$ Compute the scalar line integral

$$\int_C (x+y)\,ds$$

where C is the arc of the circle of radius 2 from $(2,0)$ to $(-2,0)$ in the counterclockwise direction, i.e. the top half.



$\vec{r}(t) = \langle 2\cos t, 2\sin t \rangle$

$\vec{r}\,'(t) = \langle -2\sin t, 2\cos t \rangle$

$\|\vec{r}\,'\| = 2$

$$\int_C (x+y)\,ds = \int_0^{\pi} (2\cos t + 2\sin t)(2)\,dt = 4\left[\sin t - \cos t\right]\Big|_0^{\pi} = 8$$

2. $\boxed{5}$ Compute the vector line integral

$$\int \langle xy, y\rangle \cdot d\mathbf{r}$$

where C is the line segment from $(1,2)$ to $(0,4)$.

parametrized the line

correctly | incorrectly

$\vec{r}(t) = \langle 1-t, 2+2t \rangle$ for $0 \le t \le 1$

$\vec{r}\,'(t) = \langle -1, 2 \rangle$

$$\int \vec{F}\cdot d\vec{r} = \int_0^1 \langle (1-t)(2+2t), (2+2t)\rangle \cdot \langle -1, 2\rangle\,dt$$

$$= \int_0^1 (2+2t)\left[(t-1)+2\right]dt = 2\int_0^1 (1+t)(t+1)\,dt$$

$$= \frac{2}{3}(1+t)^3\Big|_0^1 = \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$$

3. $\boxed{5}$ Use the Fundamental Theorem of Conservative Vector Fields to evaluate the line integral

$$\int_C \langle yz, xz, xy + z\rangle \cdot d\mathbf{r}$$

where C is given by $\mathbf{r}(t) = \left\langle \sin\left(\frac{t\pi}{4}\right), \frac{t+2}{2}, t+2 \right\rangle$ for $0 \le t \le 2$.

$$f(x,y,z) = xyz + \frac{z^2}{2}$$

$$\vec{r}(0) = \langle 0, 1, 2\rangle$$

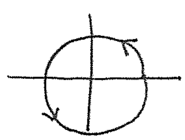
$$\vec{r}(2) = \langle 1, 2, 4\rangle$$

$$f(1,2,4) - f(0,1,2) = (8+8) - (0+2) = 14$$

4. $\boxed{5}$ Use Green's Theorem to evaluate the line integral

$$\int_C (-x^2y)\,dx + xy^2\,dy$$

where C is the boundary of the unit circle with positive orientation.



$$\int_C (-x^2y)\,dx + xy^2\,dy = \iint_D (y^2 + x^2)\,dA$$

$$= \int_0^{2\pi}\int_0^1 r^2 \cdot r\,dr\,d\theta = 2\pi\left(\frac{1}{4}\right) = \frac{\pi}{2}$$