Math 273 | Quiz 11 | Name: ______________

21 Apr 2016 | Show appropriate work. | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{5}$ Evaluate

$$\iint_S \langle x, y, z^2 \rangle \cdot d\mathbf{S}$$

where S is the cone parameterized by

$$G(u,v) = \langle 2u\cos v, 2u\sin v, u\rangle$$

for $0 \le u \le 2, 0 \le v \le 2\pi$ with outward/downward pointing normal vectors.

$\vec{T}_u = \langle 2\cos v, 2\sin v, 1 \rangle$

$\vec{T}_v = \langle -2u\sin v, 2u\cos v, 0 \rangle$

$\vec{T}_u \times \vec{T}_v = \langle -2u\cos v, -2u\sin v, \underline{\underline{4u}} \rangle$ which is <u>upward</u>

$\vec{N} = \langle 2u\cos v, 2u\sin v, -4u \rangle$ is <u>downward</u>

$\vec{F} = \langle 2u\cos v, 2u\sin v, u^2 \rangle$

$\vec{F}\cdot\vec{N} = 4u^2\cos^2 v + 4u^2\sin^2 v - 4u^3$

$= 4u^2 - 4u^3$

$$\iint_S \vec{F}\cdot d\vec{S} = \int_0^2\int_0^{2\pi} 4(u^2-u^3)\,du = 4(2\pi)\left(\frac{u^3}{3} - \frac{u^4}{4}\right)\Bigg|_0^2$$

$$= 8\pi\left(\frac{8}{3} - \frac{16}{4}\right) = \frac{-32\pi}{3}$$

2. $\boxed{5}$ Evaluate

$$\iint_S \langle xz, yz, x^2+y^2\rangle \cdot d\mathbf{S}$$

where S is the surface consisting of the cylinder $x^2+y^2=4$ for $0 \le z \le 3$ with outward pointing normal and the disk $x^2+y^2 \le 4$ in the xy-plane with downward normal. [S is a drinking glass, with normal vectors pointing outward.]

S_1: 

$G(\theta, z) = \langle 2\cos\theta, 2\sin\theta, z\rangle \qquad 0 \le \theta \le 2\pi, \ 0 \le z \le 3$

$\vec{T}_\theta = \langle -2\sin\theta, 2\cos\theta, 0\rangle$

$\vec{T}_z = \langle 0, 0, 1\rangle$

$\vec{N} = \vec{T}_\theta \times \vec{T}_z = \langle 2\cos\theta, 2\sin\theta, 0\rangle$ which points outward

$\vec{F} = \langle 2z\cos\theta, 2z\sin\theta, 4\rangle$

$\vec{F}\cdot\vec{N} = 4z\cos^2\theta + 4z\sin^2\theta = 4z$

$$\iint_{S_1} \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_0^3 4z \, dz\, d\theta = 2\pi\left(2z^2\right)\Big|_0^3 = 36\pi$$

S_2: 

$G(\theta, r) = \langle r\cos\theta, r\sin\theta, 0\rangle \qquad 0 \le r \le 2, \ 0 \le \theta \le 2\pi$

$\vec{N} = \langle 0, 0, -r\rangle$ is downward

$\vec{F} = \langle 0, 0, r^2\rangle$

$\vec{F}\cdot\vec{N} = \cancel{\langle 0, 0,} \ -r^3$

$$\iint_{S_2} \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_0^2 (-r^3)\, dr\, d\theta = 2\pi\left(\frac{-r^4}{4}\right)\Big|_0^2 = -8\pi$$

$$\text{so} \iint_S \vec{F}\cdot d\vec{S} = 36\pi - 8\pi = 28\pi$$