1. Evaluate

\[ \int \int_S \langle x, y, z^2 \rangle \cdot dS \]

where \( S \) is the cone parameterized by \( G(u, v) = (2u \cos v, 2u \sin v, u) \)

for \( 0 \leq u \leq 2, 0 \leq v \leq 2\pi \) with outward/downward pointing normal vectors.

\[ \overrightarrow{u} = \langle 2 \cos v, 2 \sin v, 1 \rangle \]
\[ \overrightarrow{v} = \langle -2u \sin v, 2u \cos v, 0 \rangle \]
\[ \overrightarrow{u} \times \overrightarrow{v} = \langle -2u \cos v, -2u \sin v, A_u \rangle \text{ which is upward} \]
\[ \vec{N} = \langle 2u \cos v, 2u \sin v, -4u \rangle \text{ is downward} \]
\[ \vec{F} = \langle 2u \cos v, 2u \sin v, u^2 \rangle \]
\[ \vec{F} \cdot \vec{N} = 4u^2 \cos^3 v + 4u^2 \sin^2 v - 4u^3 \]
\[ = 4u^2 - 4u^3 \]

\[ \int \int_S \vec{F} \cdot d\vec{S} = \int_0^2 \int_0^{2\pi} 4(u^3 - u^3) \, du \, dv = 4(2\pi) \left( \frac{u^4}{3} - \frac{u^4}{4} \right) \bigg|_0^2 = 8\pi \left( \frac{8}{3} - \frac{1}{4} \right) = -\frac{32\pi}{3} \]
2. Evaluate

\[ \iint_S \langle xz, yz, x^2 + y^2 \rangle \cdot dS \]

where \( S \) is the surface consisting of the cylinder \( x^2 + y^2 = 4 \) for \( 0 \leq z \leq 3 \) with outward pointing normal and the disk \( x^2 + y^2 \leq 4 \) in the \( xy \)-plane with downward normal. [\( S \) is a drinking glass, with normal vectors pointing outward.]

\[ \begin{align*}
S_1 & : \quad \mathbf{G}(\theta, z) = \langle 2\cos\theta, 2\sin\theta, z \rangle \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq z \leq 3 \\
\mathbf{r}_\theta & = \langle -2\sin\theta, 2\cos\theta, 0 \rangle \\
\mathbf{r}_z & = \langle 0, 0, 1 \rangle \\
\mathbf{N} & = \mathbf{r}_\theta \times \mathbf{r}_z = \langle 2\cos\theta, 2\sin\theta, 0 \rangle \quad \text{which points outward} \\
\mathbf{F} & = \langle 2z\cos\theta, 2z\sin\theta, 4 \rangle \\
\mathbf{F} \cdot \mathbf{N} & = 4z \cos^2\theta + 4z \sin^2\theta = 4z \\
\int_S \iint F \cdot dS & = \int_0^3 \int_0^{2\pi} 4z \, d\theta \, dz = 2\pi \left( 2z^2 \right) \bigg|_0^3 = 36\pi
\end{align*} \]

\[ S_2 : \quad \mathbf{G}(\theta, r) = \langle r\cos\theta, r\sin\theta, 0 \rangle \quad 0 \leq r \leq 2, \quad 0 \leq \theta \leq 2\pi \\
\mathbf{N} = \langle 0, 0, -1 \rangle \quad \text{is downward} \\
\mathbf{F} = \langle 0, 0, r^2 \rangle \\
\mathbf{F} \cdot \mathbf{N} = \langle 0, 0, -r^3 \rangle \\
\int_S \iint F \cdot dS = \int_0^2 \int_0^{2\pi} (-r^3) \, d\theta \, dr = 2\pi \left( -\frac{r^4}{4} \right) \bigg|_0^2 = -8\pi
\]

\[ S_0 \quad \int_S \iint F \cdot d\mathbf{S} = 36\pi - 8\pi = 28\pi \]