Math 273 | Quiz 12 | Name: ____________
26 Apr 2016 | Show appropriate work. | Point Values in $\boxed{\text{boxes}}$.

Today we are interested in the flux through a silo S. For the purpose of this worksheet, define a silo to be the cylinder $x^2 + y^2 = 4$ for $0 \le z \le 6$ with a hemispherical cap on the top. There is no bottom on our silo. Normal vectors point outward.

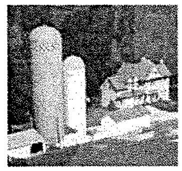


Figure 1: My grandpa's farm, complete with silos.

On this page we are interested in using The Divergence Theorem to compute

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F} = \left\langle x + \cos\left(y^2 + z^{2016}\right), y + \sin\left(x^2 + z^{2016}\right), y^2 \right\rangle$.

1. Since the Divergence Theorem requires the surface be closed, we start need to do a little work up front.

   (a) $\boxed{2}$ Let S_f be the surface representing the floor with downward normal. Compute the surface integral

   $$\iint_{S_f} \mathbf{F} \cdot d\mathbf{S}.$$

   $\vec{N} = \langle 0, 0, -r \rangle$

   $\vec{F} = \langle ?, ?, r^2 \sin^2\theta \rangle$

   So $\displaystyle\iint_{S_f} \vec{F} \cdot d\vec{S} = \int_0^{2\pi}\int_0^{2} -r^3 \sin^2\theta \, dr\, d\theta = \frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{2\pi} \left(-\frac{r^4}{4}\right)\Big|_0^{2} = \pi(-4)$

   $= -4\pi$

   (b) $\boxed{1}$ Since $S + S_f$ is now a closed surface, use the Divergence Theorem to compute

   $$\iint_{S+S_f} \mathbf{F} \cdot d\mathbf{S}.$$

   $\displaystyle = \iiint_{\text{solid silo}} 2\, dV = 2\left[\pi(4)(6) + \frac{1}{2}\cdot\frac{4}{3}\pi(2)^3\right] = 48\pi + \frac{32\pi}{3} = \frac{176\pi}{3}$

   (c) $\boxed{1}$ Subtract to evalate

   $$\iint_S \mathbf{F} \cdot d\mathbf{S}. = \frac{176\pi}{3} - (-4\pi) = \frac{188\pi}{3}$$

On this page, we are interested in using Stokes' Theorem in two different ways to compute

$$\iint_S \operatorname{curl}\mathbf{F} \cdot d\mathbf{S}.$$

Below are two vector field $\mathbf{F}$. For each, choose an appropriate way to use Stokes' Theorem and then do so to evaluate the integral.

2. $\boxed{3}$ $\mathbf{F} = \left\langle xz^{2016} - y, x + y + z\sin\left(x^{2016}\right), e^{x^2+y^2} + z^{2016}\right\rangle$

We could squish the silo into a disk, or use Stokes' to evaluate a line integral; we do it the 2^{nd} way.

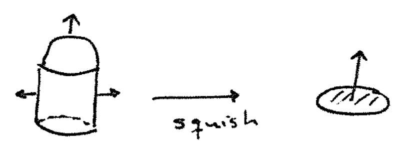
$\vec{r} = \langle 2\cos\theta, 2\sin\theta, 0\rangle \quad , \quad \vec{r}\,' = \langle -2\sin\theta, 2\cos\theta, 0\rangle$

$\vec{F}(\vec{r}) = \langle -2\sin\theta, 2\cos\theta + 2\sin\theta, e^4\rangle$

$\vec{F}\cdot\vec{r}\,' = 4\sin^2\theta + 4\cos^2\theta + 4\cos\theta\sin\theta = 4 + 4\cos\theta\sin\theta$

$$\int_{\partial S} \vec{F}\cdot d\vec{r} = \int_0^{2\pi} (4 + 4\cos\theta\sin\theta)\, d\theta = 4\theta + 2\sin^2\theta \Big|_0^{2\pi} = 8\pi$$

3. $\boxed{3}$ $\mathbf{F} = \langle yz^2 + zx^{2016}, xy^2 + y^{4.26} + ze^z, x^2y^2 - xyz\rangle$



$\operatorname{curl}(\vec{F}) = \langle \text{mess}, \text{mess}, y^2 - z^2\rangle \qquad \vec{N} = \langle 0, 0, r\rangle$ for the disk

$\operatorname{curl}(\vec{F})\cdot\vec{N} = y^2 r = r^3\sin^2\theta$ using the std. param of the disk,

i.e. $\langle r\cos\theta, r\sin\theta, 0\rangle$

$$\iint_{S_f} \operatorname{curl}(\vec{F})\cdot d\vec{S} = \int_0^{2\pi}\int_0^2 r^3\sin^2\theta\, dr\, d\theta = 4\pi$$

see front