

1. [2] Describe the given set in cylindrical coordinates and in spherical coordinates.

$$x^2 + y^2 + z^2 \leq 4, \quad x \geq y, \quad z \geq 0$$

$$r^2 + z^2 \leq 4$$



Cylindrical

$$z \leq \sqrt{4 - r^2}$$

$$-\frac{3\pi}{4} \leq \theta \leq \frac{\pi}{4}$$

Spherical

$$\rho \leq 2$$

$$-\frac{3\pi}{4} \leq \theta \leq \frac{\pi}{4}$$

$$\phi \leq \frac{\pi}{2}$$

2. [2] Parameterize the intersection of the surfaces $x^2 + z^2 = 4$ and $z = y^3$.

$$\text{so } y = \sqrt[3]{z}$$

$$\langle 2 \cos \theta, (2 \sin \theta)^{1/3}, 2 \sin \theta \rangle$$

3. [3] Find a parameterization of the tangent line to $\mathbf{r}(t) = \langle t^2, \cos \pi t, e^{1-t} \rangle$ at $t = 1$.

$$\mathbf{r}(1) = \langle 1, -1, 1 \rangle$$

$$\mathbf{r}'(t) = \langle 2t, -\pi \sin \pi t, -e^{1-t} \rangle$$

$$\mathbf{r}'(1) = \langle 2, 0, -1 \rangle$$

$$\mathbf{l}(t) = \langle 1, -1, 1 \rangle + t \langle 2, 0, -1 \rangle$$

4. [3] Find the length of the curve $\mathbf{r}(t) = \langle \cos 3t, 2t^{3/2}, \sin 3t \rangle$ over the interval $0 \leq t \leq 2\pi$.

$$\mathbf{r}'(t) = \langle -3 \sin 3t, 3t^{1/2}, 3 \cos 3t \rangle$$

$$\|\mathbf{r}'(t)\| = (9 \sin^2 3t + 9t + 9 \cos^2 3t)^{1/2} = 3\sqrt{1+t}$$

$$s = \int_0^{2\pi} 3(1+t)^{1/2} dt = 2(1+t)^{3/2} \Big|_0^{2\pi} = 2[(1+2\pi)^{3/2} - 1]$$