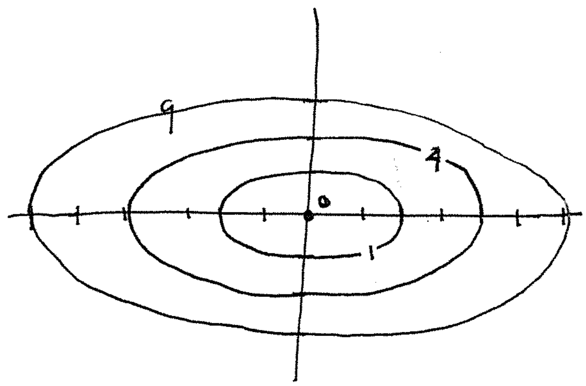
Math 273 | Quiz 5 | Name: ____________
19 Feb 2016 | Show appropriate work. | Scaled to 10 points.

1. $\boxed{3}$ Sketch the level curves corresponding to $c = 0, 1, 4$, and 9 for $f(x, y) = (x/2)^2 + y^2$.



2. $\boxed{7}$ Find the limit or show that it does not exist.

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$\lim_{(x,y)\to(0,0)} \tan(x^2+y^2)\tan^{-1}\left(\frac{1}{x^2+y^2}\right)$

2 (a) $\lim_{(x,y)\to(1,0)} e^{y^2-x}\arctan\left(\frac{x}{y^2}\right) = e^{-1}\left(\frac{\pi}{2}\right)$

(b) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ [HINT: Consider two paths, lines will do.]

5

Along $y=x$ $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+x^2} = \frac{1}{2}$

Along $x=0$ $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{0}{0+y^2} = 0$.

Since the limits are different along different paths, the limit does not exist.

3. Compute the specified partial derivatives.

(a) $\boxed{2}$ $f(x,y,z) = x^2 + \sin(x^2y + yz^2) + e^{4z}$, $\quad f_x$

$$f_x = 2x + \cos(x^2y + yz^2)(2xy)$$

(b) $\boxed{3}$ $f(x,y,z) = \dfrac{x^2y^3}{z} + \sin\left(\dfrac{y^2+e^z}{y^2+z^2+1}\right)$, $\quad f_{yzxzy}$

$$f_x = \frac{2xy^3}{z}$$

$$f_{xyy} = \frac{2x\cdot 3\cdot 2\cdot y}{z}$$

so by Clairaut's Thm

$$f_{xyyzz} = \frac{12xy(-1)(-2)}{z^3}$$

$$f_{yzxzy} = \frac{24xy}{z^3}$$

4. $\boxed{5}$ Find an equation for the tangent plane to $f(x,y) = x^2 + \sin \cancel{z}y$ at $(1,0)$.

$f_x = 2x$

$f_y = \cos y$

$f(1,0) = 1+0 = 1$

$f_x(1,0) = 2$

$f_y(1,0) = 1$

so

$$z - 1 = 2(x-1) + 1(y-0)$$

or

$$z = 2x + y - 1$$