Math 273 | Quiz 6 | Name: ______________
26 Feb 2016 | Show appropriate work. | Scaled to 10 points.

1. True / False. Assume f is a differentiable function.

   (a) $\boxed{1}$ **T** / **F** : All tangent lines to f at (a,b) are contained in the tangent plane of f at (a,b).

   (b) $\boxed{1}$ **T** / **F** : $\nabla f(a,b)$ is normal to the tangent plane of f at (a,b).

   (c) $\boxed{1}$ **T** / **F** : $\nabla f(a,b)$ is the maximum slope of f at (a,b).

2. $\boxed{5}$ Use the linear approximation of $f(x,y) = \sqrt{x^2+y^2}$ at $(3,4)$ to estimate $\sqrt{3.1^2+3.9^2}$.

$f(3,4) = 5$

$f_x = \dfrac{x}{\sqrt{x^2+y^2}}$ $\quad \Delta x = 0.1$ $\quad \sqrt{3.1^2+3.9^2} \approx f(3,4) + f_x(3,4)\,\Delta x + f_y(3,4)\,\Delta y$

$f_x(3,4) = \frac{3}{5}$ $\quad \Delta y = -0.1$ $\quad 5 + \frac{3}{5}(0.1) + \frac{4}{5}(-0.1) = 4.98$

$f_y(3,4) = \frac{4}{5}$

3. $\boxed{4}$ What is the slope of $f(x,y) = x^2 + xy + 2y$ in the direction of $\mathbf{v} = \langle 1,2\rangle$ at $(-1,0)$.

$\nabla f = \langle 2x+y,\ x+2\rangle$

$\nabla f(-1,0) = \langle -2, 1\rangle$

$\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{\langle 1, 2\rangle}{\sqrt{5}}$ $\quad D_{\vec{u}} f(-1,0) = \langle -2, 1\rangle \cdot \dfrac{\langle 1,2\rangle}{\sqrt{5}} = 0$

4. Let $\mathbf{F}(x,y,z) = \left\langle \frac{x}{y}, \frac{y}{z}, \frac{z}{x} \right\rangle$, compute the following.

   (a) $\boxed{4}$ div($\mathbf{F}$) $= \nabla \cdot \vec{F} = \frac{1}{y} + \frac{1}{z} + \frac{1}{x}$

   (b) $\boxed{4}$ curl($\mathbf{F}$) $= \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{y} & \frac{y}{z} & \frac{z}{x} \end{vmatrix} = \left\langle \frac{y}{z^2}, \frac{z}{x^2}, \frac{x}{y^2} \right\rangle$