Math 273 | Quiz 7 | Name: ____________

4 Mar 2016 | Show appropriate work. | Scaled to 10 points.

1. Consider the function

$$f(x,y) = 4x + 6y - x^2 - y^2.$$

(a) $\boxed{2}$ Find all critical points and use the Second Derivative Test to classify them.

$f_x = 4 - 2x$ The only crit pt is $(2,3)$

$f_y = 6 - 2y$ $D = f_{xx} f_{yy} - f_{xy}^2 = (-2)(-2) - (0)^2 = 4$, $f_{xx} < 0$

so $f(2,3) = 13$ is a local max

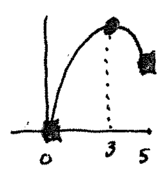
(b) Let $D = \{(x,y) | 0 \le x \le 4, 0 \le y \le 5\}$. Find all extreme values on the boundary of $D$. Please organize your work as follows.

i. $\boxed{2}$ The component of $\partial D$ with $x = 0$. for $0 \le y \le 5$

$F_1(y) = f(0,y) = 6y - y^2$

$F_1'(y) = 6 - 2y$

$F_1' = 0$ when $y = 3$




min $(0,0,0)$

max $(0,3,9)$

$(0,5,5)$

ii. $\boxed{2}$ The component of $\partial D$ with $x = 4$.

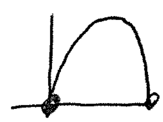
$F_2(y) = f(4,y) = 6y - y^2$, see above

min $(4,0,0)$

max $(4,3,9)$

iii. $\boxed{2}$ The component of $\partial D$ with $y = 0$.

$F_3(x) = f(x,0) = 4x - x^2$

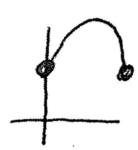


min $(0,0,0)$ & $(4,0,0)$

max $(2,0,4)$

iv. $\boxed{2}$ The component of $\partial D$ with $y = 5$.

$F_4(x) = f(x,5) = 4x - x^2 + 5$



min $(0,5,5)$ & $(4,5,5)$

max $(2,5,9)$

(c) $\boxed{2}$ Find the absolute minimum and absolute maximum of $f(x,y)$ on $D$.

Absolute Max of 13 @ $(2,3)$

Absolute Min of 0 @ $(0,0)$ & $(4,0)$

2. $\boxed{8}$ Use the method of Lagrange multipliers to find the maximum volume of a rectangular box that can be inscribed (with edges parallel to the coordintate axes) in the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$$

$V = 8xyz$

$g(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (3)

$\nabla V = \langle 8yz, 8xz, 8xy \rangle$

$\nabla g = \langle \frac{2}{a^2}x, \frac{2}{b^2}y, \frac{2}{c^2}z \rangle$

$\nabla V = \lambda \nabla g$

$8yz = \lambda \frac{2}{a^2}x \longrightarrow \lambda = \frac{4yz}{x}a^2$

$8xz = \lambda \frac{2}{b^2}y \longrightarrow \lambda = \frac{4xz}{y}b^2$

$\left.\right\}$ $z = 0$ or $\frac{x^2}{a^2} = \frac{y^2}{b^2}$ (1)

$8xy = \lambda \frac{2}{c^2}z \longrightarrow \lambda = \frac{4xy}{z}c^2$

Similarly,

$\frac{x^2}{a^2} = \frac{z^2}{c^2}$ (2)

Substituting (1) & (2) into (3)

gives $\frac{x^2}{a^2} = \frac{y^2}{b^2} = \frac{z^2}{c^2} = \frac{1}{3}$

So $x = \frac{a}{\sqrt{3}}$, $y = \frac{b}{\sqrt{3}}$, $z = \frac{c}{\sqrt{3}}$

$V = \frac{8abc}{3\sqrt{3}}$