Math 273
1 Apr 2016

Quiz 9
Show appropriate work.

Name: _____
Point Values in boxes.

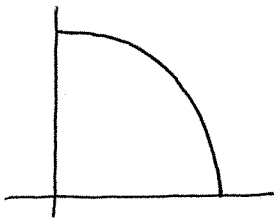
1. [5] Set up an integral in spherical coordinates representing the volume inside the sphere of radius 1, outside the cone $z = \sqrt{x^2 + y^2}$, and above the xy-plane. **Do not evaluate the integral.**

$\phi = \frac{\pi}{4}$

$$\int_0^{2\pi} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^1 \underbrace{\rho^2 \sin\phi}_{+1} \, d\rho \, d\phi \, d\theta = 2\pi \left(-\cos\phi \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} \right) \frac{1}{3}$$

$$= 2\pi \left(\frac{\sqrt{2}}{2} \right) \left(\frac{1}{3} \right) = \frac{\sqrt{2}\pi}{3}$$

2. [5] Find the centroid of the quarter circle of radius R centered at the origin in the first quadrant using polar coordinates.



$$A_{area} = \frac{1}{4}\pi R^2$$

$\bar{x} = \bar{y}$ by symmetry

$$\bar{x} = \frac{4}{\pi R^2} \int_0^{\frac{\pi}{2}} \int_0^R \underbrace{r\cos\theta}_{x} \underbrace{r \, dr \, d\theta}_{dA} = \frac{4}{\pi R^2} \int_0^{\frac{\pi}{2}} \cos\theta \, d\theta \int_0^R r^2 \, dr$$

$$= \frac{4}{\pi R^2} \cdot \sin\theta \Big|_0^{\frac{\pi}{2}} \cdot \frac{r^3}{3} \Big|_0^R$$

$$= \frac{4}{\pi R^2} \cdot 1 \cdot \frac{R^3}{3} = \frac{4R}{3\pi}$$