Math 274 Section:_____ Name:__________________

Exam 2 - Friday, 3 November 2017

Show all appropriate work to receive full credit. Point values are in $\boxed{\text{boxes}}$

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at Montana State University, and that I have neither given nor received any unauthorized assistance (calculator, cell phone, notes, other people's exams, etc.) on this exam. Furthermore, I have and will not discuss the content of this exam, or provide any written or verbal aid to others. I understand that if I violate this statement, I will receive an exam score of 0." SIGNATURE:______________________________

1. Determine the Laplace transform of the following functions.

   (a) $\boxed{5}$ $f(t) = 2e^{3t}\cos 4t$

   $$F(s) = \frac{2(s-3)}{(s-3)^2+4^2}$$

   (b) $\boxed{5}$ $g(t) = \int_0^t \sin(2t-2v)\cos 2v\,dv$ $= \sin 2t * \cos 2t$

   $$G(s) = \frac{2}{s^2+4}\cdot\frac{s}{s^2+4}$$

2. $\boxed{8}$ Express the following function using step functions and determine its Laplace transform.

$$m(t) = \begin{cases} e^t, & 0 < t < 2 \\ t, & 2 < t \end{cases}$$

$$= e^t + u(t-2)\,(t - e^t)$$

$$M(s) = \frac{1}{s-1} + e^{-2s}\mathcal{L}\{(t+2) - e^{t+2}\}$$

$$= \frac{1}{s-1} + e^{-2s}\left(\frac{1}{s^2} + \frac{2}{s} - \frac{e^2}{s-1}\right)$$

3. Determine the inverse Laplace transform of the following functions.

(a) $\boxed{7}$ $$G(s) = \frac{3s}{s^2+6s+13} = \frac{3(s+3) - 9}{(s+3)^2 + 2^2}$$

$$g(t) = 3e^{-3t}\cos 2t - \frac{9}{2}e^{-3t}\sin 2t$$

(b) $\boxed{8}$ $$H(s) = \frac{s}{s^2+1} + e^{-\pi s}\left(\frac{s-1}{s^2+1}\right)$$

$$h(t) = \cos t + u(t-\pi)\left(\cos(t-\pi) - \sin(t-\pi)\right)$$

4. $\boxed{8}$ Assume $g(t)$ is piecewise continuous on $[0,\infty)$ and of exponential order. Find a formula for the solution to

$$y' + y = g(t), \qquad y(0) = 0.$$

You may leave your solution $y(t)$ in terms of a convolution.

$$sY + Y = G(s)$$

$$Y = G \cdot \frac{1}{s+1}$$

$$y(t) = g(t) * e^{-t}$$

5. $\boxed{12}$ Applying the Laplace transform to the initial value problem

$$y'' - 6y' + 9y = e^{2t}, \qquad y(0) = 3, y'(0) = 4$$

gives the following

$$Y(s) = \frac{3s^2 - 20s + 29}{(s-2)(s-3)^2}. \tag{1}$$

Determine $y(t) = \mathscr{L}^{-1}\{Y(s)\}$, the solution to the given initial value problem.

$$\frac{3s^2 - 20s + 29}{(s-2)(s-3)^2} = \frac{A}{s-2} + \frac{B}{s-3} + \frac{C}{(s-3)^2}$$

$$3s^2 - 20s + 29 = A(s-3)^2 + B(s-2)(s-3) + C(s-2)$$

Let $s=2$: $12 - 40 + 29 = A \Rightarrow A = 1$

$s = 3$: $27 - 60 + 29 = C \Rightarrow C = -4$

Eq. Coeff

s^2 : $3 = A + B$ so $B = 2$

$$y(t) = e^{2t} + 2e^{3t} - 4te^{3t}$$

6. $\boxed{10}$ Apply the Laplace transform to the initial value problem

$$y'' + 3y' = 7, \qquad y(0) = 1, y'(0) = -3$$

to express $Y(s) = \mathscr{L}\{y(t)\}$ in the form $Y(s) = \frac{P(s)}{Q(s)}$; for example, (1) above is of this form. **Do not find the inverse Laplace transform.**

$$s^2 Y - s + 3 + 3Y - 3 = \frac{7}{s}$$

$$Y(s^2 + 3) = \frac{7 + s^2}{s}$$

$$Y = \frac{7 + s^2}{s(s^2+3)}$$

a

7. Consider a mass-spring system sitting in front of cuckoo clock. After π seconds the time is exactly 1 pm. The cuckoo comes out of the clock and strikes the system exerting an impulse on the mass. The system is governed by the symbolic initial value problem

$$x'' + 4x = 2\delta(t - \pi), \qquad x(0) = 0, x'(0) = -2, \tag{2}$$

where $x(t)$ measures the displacement from the equilibrium.

(a) $\boxed{8}$ Determine $x(t)$, i.e. solve the symbolic initial value problem (2).

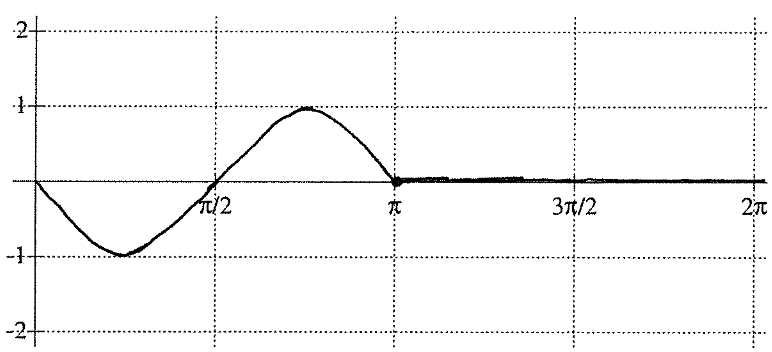
$$s^2 X + 2 + 4X = 2e^{-\pi s}$$

$$X = \frac{2e^{-\pi s} - 2}{s^2 + 4}$$

$$x = u(t-\pi)\sin(2t - 2\pi) - \sin(2t)$$

$$= u(t-\pi)\sin 2t - \sin 2t$$

(b) $\boxed{4}$ Carefully sketch a graph of $x(t)$ for $t \in [0, 2\pi]$. [HINT: sine is periodic.]



Problem	1	2	3	4	5	6	7	Total
Value	10	8	15	8	12	10	12	75
Points								