Math 274 — Inverse Laplace Thing — Name: ______________

18 Oct 2017 — Show Appropriate Work — Scaled to $\boxed{10}$.

1. Determine the inverse Laplace transform of the following.

(a) $\boxed{1}$ $F(s) = \dfrac{3}{s+2}$

$f(t) = 3e^{-2t}$

(d) $\boxed{1}$ $J(s) = \dfrac{3}{(s+2)^6} = \dfrac{3}{5!} \cdot \dfrac{5!}{(s+2)^6}$

$j(t) = \dfrac{3}{5!} e^{-2t} t^5$

(b) $\boxed{1}$ $G(s) = \dfrac{3}{2s+2}$

$g(t) = \dfrac{3}{2} e^{-t}$

(e) $\boxed{1}$ $K(s) = \dfrac{3}{s^2+2}$

$K(t) = \dfrac{3}{\sqrt{2}} \sin(\sqrt{2}t)$

(c) $\boxed{1}$ $H(s) = \dfrac{3}{4-s}$

$h(t) = -3e^{4t}$

(f) $\boxed{1}$ $M(s) = \dfrac{3s}{s^2+2}$

$m(t) = 3\cos(\sqrt{2}t)$

(g) $\boxed{2}$ $N(s) = \dfrac{3}{s^2+2s} = \dfrac{A}{s} + \dfrac{B}{s+2}$

so $3 = A(s+2) + Bs$

Let $s=0 \Rightarrow A = \frac{3}{2}$

Let $s=-2 \Rightarrow B = -\frac{3}{2}$

$n(t) = \dfrac{3}{2} - \dfrac{3}{2} e^{-2t}$

(h) $\boxed{2}$ $P(s) = \dfrac{3s}{s^2+2s+5} = \dfrac{3(s+1)-3}{(s+1)^2+2^2}$

$p(t) = 3e^{-t}\cos 2t - \frac{3}{2} e^{-t} \sin 2t$

2. $\boxed{2}$ Last time we noted that since $\mathscr{L}\{tf(t)\} = -F'(s)$ we had $f(t) = \frac{-1}{t}\mathscr{L}^{-1}\{F'(s)\}$. Using this, determing the inverse Laplace transform of

$$F(s) = \ln\left(\frac{s^2+1}{s^2-s-6}\right). = \ln(s^2+1) - \ln(s-3) - \ln(s+2)$$

so $F'(s) = \frac{2s}{s^2+1} - \frac{1}{s-3} - \frac{1}{s+2}$

so $f(t) = -\frac{1}{t}\left(2\cos t - e^{3t} - e^{-2t}\right)$

3. $\boxed{8}$ Applying the Laplace transform to the initial value problem

$$y'' - 2y' + 17y = 17e^{2t}, \qquad y(0) = 4, y'(0) = 1$$

gives the following

$$\left[s^2Y(s) - 4s - 1\right] - 2\left[sY(s) - 4\right] + 17Y(s) = \frac{17}{s-2}. \tag{1}$$

Solve equation (1) above for $Y(s)$ and then determine $y(t) = \mathscr{L}^{-1}\{Y(s)\}$ which is the solution to the given initial value problem.

$$Y(s)\left[s^2 - 2s + 17\right] = \frac{17}{s-2} + 4s - 7 = \frac{17 + 4s^2 - 15s + 14}{s-2} = \frac{4s^2 - 15s + 31}{s-2}$$

$$Y(s) = \frac{4s^2 - 15s + 31}{(s-2)(s^2-2s+17)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+17}$$

$$4s^2 - 15s + 31 = A(s^2-2s+17) + (Bs+C)(s-2)$$

Let $s = 2$: $16 - 30 + 31 = A(17) \Rightarrow A = 1$

Eq Coeff : s^2 : $4 = A + B$ so $B = 3$

s^0 : $31 = 17 - 2C$ so $C = -7$

$$Y(s) = \frac{1}{s-2} + \frac{3s-7}{(s-1)^2+4^2} = \frac{1}{s-2} + \frac{3(s-1)}{(s-1)^2+4^2} - \frac{4}{(s-1)^2+4^2}$$

$$y(t) = e^{2t} + 3e^t\cos 4t - e^t \sin 4t$$