Math 274 | Laplace Thing 1 | Name: ____________
13 Oct 2017 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

Hopefully you will finish this in class, if not, it will be due at the beginning of class on Monday.

1. $\boxed{3}$ Use the definition to compute $\mathscr{L}\{t\}$.

$$\mathcal{L}\{t\} = \int_0^\infty te^{-st}\,dt = -\frac{t}{s}e^{-st}\Big|_0^\infty + \frac{1}{s}\int_0^\infty e^{-st}\,dt$$

$u = t \qquad dv = e^{-st}\,dt$

$du = dt \qquad v = -\frac{1}{s}e^{-st}$

[You are using L'Hopital's Rule about here, right?]

$$= \lim_{N\to\infty}\left[\frac{-N}{s}e^{-sN} + 0\right] + \lim_{N\to\infty}\left[-\frac{e^{-st}}{s} + \frac{1}{s^2}\right]$$

$$\underline{\text{Note}}: \lim_{N\to\infty}\frac{-N}{se^{+sN}} \overset{L'H}{=} \lim_{N\to\infty}\frac{-1}{+s^2e^{+sN}} = 0 \quad \text{for } s>0$$

$$= \frac{1}{s^2} \quad \text{for } s>0$$

2. $\boxed{2}$ Use Table 7.1 in your text and linearity to determine the Laplace transform of the following functions.

   (a) $f(t) = 4t^2 - 7e^{2t}\cos 3t$

$$F(s) = 4\cdot\frac{2}{s^3} - 7\cdot\frac{s-2}{(s-2)^2+3^2}$$

   (b) $g(t) = 6 + e^{-3t}t^8$

$$G(s) = \frac{6}{s} + \frac{8!}{(s+3)^9}$$

3. $\boxed{2}$ For each of the following choose all that apply; the function is not piecewise continuous (**NPC**), the function is not of exponential order (**NEO**), and/or the function has a Laplace transform (**LT**).

(a) $\boxed{\textbf{NPC}}$ / $\boxed{\textbf{NEO}}$ / **LT** : $f(t) = \sec t$

(b) **NPC** / **NEO** / $\boxed{\textbf{LT}}$: $f(t) = e^{\sin t}$

(c) **NPC** / $\boxed{\textbf{NEO}}$ / **LT** : $f(t) = 14e^{t^2+7}$

(d) **NPC** / **NEO** / $\boxed{\textbf{LT}}$: $f(t) = \begin{cases} 4e^t, & 0 < t < 2 \\ 12e^t, & 2 < t \end{cases}$

4. $\boxed{3}$ The inverse Laplace transform is defined as you would expect it to be[1]. For example, since $\mathscr{L}\{\sin 3t\} = \dfrac{3}{s^2+9}$, we have $\mathscr{L}^{-1}\left\{\dfrac{3}{s^2+9}\right\} = \sin 3t$. Find the inverse Laplace transform of the following.

(a) $F(s) = \dfrac{s}{s^2+4}$

$f(t) = \cos 2t$

(b) $G(s) = \dfrac{8s}{s^2+4}$

$g(t) = 8\cos 2t$

(c) $H(s) = \dfrac{6}{s^4}$

$h(t) = t^3$

(d) $K(s) = \dfrac{1}{s^4}$

$k(t) = \dfrac{t^3}{6}$

(e) $W(s) = \dfrac{6-7s}{s^2+9} = \dfrac{3\cdot 2}{s^2+3^2} - \dfrac{7\cdot s}{s^2+3^2}$ [Hint: separate the fraction.]

$w(t) = 2\sin 3t - 7\cos 3t$

[1]There are some technical complications we will address next week, but for now we will ignore them and assume it works as it should.