Math 274 Quiz 6 Name: ____________

6 Oct 2017 Show Appropriate Work Point Values in $\boxed{\text{boxes}}$.

Variation of Parameters

If y_1 and y_2 are linearly independent solutions to $y'' + p(t)y' + q(t)y = 0$, then a particular solution to $y'' + p(t)y' + q(t)y = g(t)$ is given by

$$y_p(t) = y_1(t) \int \frac{-g(t)y_2(t)}{W[y_1, y_2](t)}\,dt + y_2(t) \int \frac{g(t)y_1(t)}{W[y_1, y_2](t)}\,dt.$$

1. $\boxed{10}$ Find a particular solution to

$$t^2y'' + ty' = t^{-3}.$$

$r^2 = 0$ so $y_1 = t^0 = 1$ $y_2 = t^0 \ln t = \ln t$

$$W = \begin{vmatrix} 1 & \ln t \\ 0 & \frac{1}{t} \end{vmatrix} = \frac{1}{t}$$

St. Form $y'' + \frac{y'}{t} = t^{-5}$.

$$v_1 = \int \frac{-t^{-5}\ln t}{t^{-1}}\,dt = \int -t^{-4}\ln t\,dt = \frac{t^{-3}}{3}\ln t - \int \frac{t^{-4}}{3}\,dt = \frac{t^{-3}}{3}\ln t + \frac{t^{-3}}{9}$$

$u = \ln t$ $dv = -t^{-4}$

$du = \frac{1}{t}dt$ $v = \frac{t^{-3}}{3}$

$$v_2 = \int \frac{t^{-5}}{t^{-1}}\,dt = \int t^{-4}\,dt = \frac{t^{-3}}{-3}$$

so $y_p = \frac{t^{-3}}{3}\ln t + \frac{t^{-3}}{9} + \frac{t^{-3}}{-3}\ln t = \frac{t^{-3}}{9} = \frac{1}{9t^3}$