C MUCs

1. $y'' + 4y = 0 \qquad r^2 + 4 = 0 \qquad r = \pm 2i \qquad$ so $\quad y_h = C_1 \cos 2t + C_2 \sin 2t$

2. $y'' + 4y = 8\cos 2t$

Try $\quad y_p = At\cos 2t + Bt \sin 2t$

$y_p' = (A + 2Bt)\cos 2t + (B - 2At)\sin 2t$

$y_p'' = (4B - 4At)\cos 2t + (-4A - 4Bt)\sin 2t$

Substituting yields

$y_p'' + 4y_p = [4B - 4At + 4At]\cos 2t + [-4A - 4Bt + 4Bt]\sin 2t$

$= 4B\cos 2t - 4A\sin 2t$

$= 8\cos 2t \qquad$ so $\quad B = 2, \quad A = 0$

$y_p = 2t\sin 2t$

3. Consider $z'' + 4z = 8e^{2it} \qquad$ Note: $\mathrm{Re}(8e^{2it}) = 8\cos 2t$

Try $\quad z_p = Ate^{2it}$

$z_p' = (A + 2iAt)e^{2it}$

$z_p'' = (4Ai - 4At)e^{2it}$

Substituting yields

$z_p'' + 4z_p = [4Ai - 4At + 4At]e^{2it}$

$= 4Aie^{2it} = 8e^{2it}$

so $\quad A = \frac{8}{4i} = \frac{2}{i} = -2i$

$y_p = \mathrm{Re}(z_p) = \mathrm{Re}[-2it\,e^{2it}] = 2t\sin 2t$

4. $y = C_1\cos 2t + C_2\sin 2t + 2t\sin 2t$