Math 274 | Quiz 1 | Name: ______________

1 Sept 2017 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{4}$ Verify that $\phi(x) = \dfrac{2}{1 - ce^x}$, where c is an arbitrary constant, is a one-parameter family of solutions to

$$\frac{dy}{dx} = \frac{y(y-2)}{2}.$$

$$\varphi' = -2(1-ce^x)^{-2}(ce^x) = \frac{2ce^x}{(1-ce^x)^2}$$

$$\frac{\varphi(\varphi-2)}{2} = \frac{1}{2}\left(\frac{2}{1-ce^x}\right)\left(\frac{2}{1-ce^x} - 2\right) = \frac{2-2(1-ce^x)}{(1-ce^x)^2} = \frac{2ce^x}{(1-ce^x)^2}$$

Since $\varphi' = \dfrac{\varphi(\varphi-2)}{2}$, $\varphi = \dfrac{2}{1-ce^x}$ is a family of solutions to the equation

2. Consider the differential equation

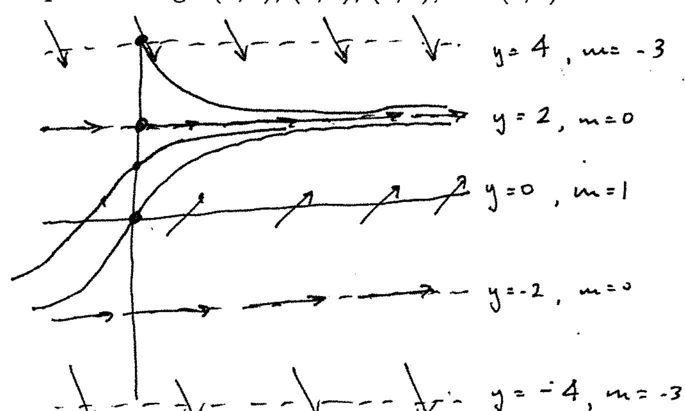
$$\frac{dy}{dx} = \frac{4 - y^2}{4}.$$

(a) $\boxed{3}$ Sketch the direction field for the equation. Include a few isoclines with their corresponding direction arrows; hint, the isoclines corresponding to slopes of 0, 1, and -3 are easy to work with, use those. Sketch the solution curves that pass through $(0,0)$, $(0,1)$, $(0,2)$, and $(0,4)$.

$\frac{dy}{dx} = 0$ when $\frac{4-y^2}{4} = 0$, i.e. $y = \pm 2$

$\frac{dy}{dx} = 1$ when $\frac{4-y^2}{4} = 1$, i.e. $y = 0$

$\frac{dy}{dx} = -3$ when $\frac{4-y^2}{4} = -3$, i.e. $y = \pm 4$



(b) $\boxed{3}$ Since the equation is autonomous we can consider its phase line. Sketch the the phase line for the equation. Classify each equilibrium.